CSC165 fall 2017

Mathematical expression: predicate logic

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Using Course notes: Mathematical Expression: predicate logic
Outline

- bi-implication
- predicates
- quantifiers
- multiple quantifiers
- mixed quantifiers
- negation
- number theory intro
- notes
- annotated slides
compare and contrast...

“If it rains, then I will wear sneakers.”
“If and only if it rains, then I will wear sneakers.”

\[(r \Rightarrow s) \iff (r \Leftrightarrow s)\]

\[r \land s \implies T \iff T\]
\[r \land \neg s \implies F \iff F\]
\[\neg r \land s \implies T \iff T\]
\[\neg r \land \neg s \implies T \iff T\]
what's a predicate?

\[ n > 7.2 \]
\[ x \text{ is tall} \]

\[ n = 5 \rightarrow F \]
\[ n = 11 \rightarrow T \]
\[ n = "Fred" \rightarrow \text{WTF???} \]
\[ \text{Danny is tall} \rightarrow F \]
\[ x = \text{KK} \rightarrow T \]
\[ x = 7 \rightarrow ?? ? \]

Boolean function \( \rightarrow \mathbb{E}, \mathbb{F} \)

Domain

\[ n > 7.2 : \mathbb{R} \rightarrow \mathbb{E}, \mathbb{F} \]

\( x \text{ is tall} : \text{Animal} \rightarrow \mathbb{E}, \mathbb{F} \)
predicate definitions — naming, use symbols

IsTall: “x is tall”, where x is an animal

G(n): “n > 7.2”, where n ∈ N
quantifiers $\forall$ and $\exists$

$n > 7.2$ depends on $n$

$\forall n \in \mathbb{N}, n > 7.2$

means $0 > 7.2 \lor 1 > 7.2 \lor 2 > 7.2 \lor 3 > 7.2 \lor \cdots$

False because counterexample $0$ is not $> 7.2$ makes entire $\lor \cdots \lor$ False.

$\exists n \in \mathbb{N}, n > 7.2$

means $0 > 7.2 \lor 1 > 7.2 \lor 2 > 7.2 \lor \cdots$

True - just need one example, e.g. $8 > 7.2$.
translate quantified predicates

\[ \forall n \in \mathbb{N}, \ n > 7.2 \]
For every natural number \( n \), \( n \) is greater than 7.2.

Every natural number is greater than 7.2.

\[ \exists n \in \mathbb{N}, \ n > 7.2 \]
There exist some natural number \( n \),
where (such that) \( n \) is greater than 7.2.

There is a natural number greater than 7.2.
quantified binary predicates

\[ x + y = 17 : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}, \mathbb{F}^3 \]

\[ \forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x + y = 17 \]

Equivalent

\[ \forall x, y \in \mathbb{Z}, x + y = 17 \]

False, e.g. \( x = 3, y = 4 \), \( x + y \neq 17 \)

Because both quantifiers are \( \forall \) (same)

Means same as \( \forall y, \forall x \in \mathbb{Z}, x + y = 17 \)

\[ \forall y, \forall x \in \mathbb{Z}, x + y = 17 \]

\[ \exists x, y \in \mathbb{Z}, x + y = 17 \]

True \( \exists x = 16, y = 1 \)

\[ \exists y \in \mathbb{Z}, \exists x \in \mathbb{Z}, x + y = 17 \]
multiple quantifier examples

done

order matters!

\[ x + y = 17 \]

\[ \forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x + y = 17 \]

True, read from left to right, then choose/pick

fix arbitrary \( x \), then \( x + y = x + (17 - x) = 17 \)

\[ y = 17 - x \]

\[ \exists y \in \mathbb{Z}, \forall x \in \mathbb{Z}, \ x + y = 17 \]

False, \( y \) is chosen before \( x \), this claims there is some \( y \) with

\[ 0 + y = 17 \]

\[ 1 + y = 17 \]

\[ -1 + y = 17 \]