This test consists of 3 questions on 6 pages (including this one). When you receive the signal to start, please make sure that your copy of the test is complete.

Please answer questions in the space provided. You will earn 20% for any question you leave blank or write “I cannot answer this question,” on.

Good Luck!
Question 1.  [22 marks]

Consider the following Python functions and definitions, where \( S, S1 \) are sets and \( P, P1-P4 \) are boolean functions. Note: feel free to ask about how Python works, since you are being tested on logic, not programming.

```python
def q0(S, P):
    return not all({P(x) for x in S})
def q1(S, P):
    return any({P(x) for x in S})
def q2(S, P):
    return not any({P(x) for x in S})
def q3(S, P):
    return all({P(x) for x in S})
```

\( S1 = \{0, 1, 2, 3, 4, 5\} \)

```python
def P1(x):
    return x > 2
def P2(x):
    return x > 3
def P3(x):
    return P1(x) and P2(x)
def P4(x):
    return (not P2(x)) or P1(x)
```

Part (a)  [6 marks]

Write the name of each function \( q0--q3 \) beside the comment(s) below that best describes the condition for which the function returns True. Indicate which are negations of each other.

1. \( \forall x \in S, P(x) \)  \( q3 \)
2. \( \exists x \in S, P(x) \)  \( q1 \)
3. \( \forall x \in S, \neg P(x) \)  \( q2 \)
4. \( \exists x \in S, \neg P(x) \)  \( q0 \)
Part (b)  [16 marks]

Use your answer for the previous part to predict what the output is below. For each answer, briefly explain your thinking.

1. \( q_0(S_1, P_2) \)  
   True  
   \( \exists x \in S_1, \neg(x > 3) \rightarrow 0, 1, 2, 3 \) are examples.

2. \( q_1(S_1, P_1) \)  
   True  
   \( \exists x \in S_1, x > 2 \rightarrow \text{true}, 3, 4, 5 \) are examples.

3. \( q_2(S_1, P_2) \)  
   False  
   \( \forall x \in S_1, \neg(x > 3) \rightarrow 4 \) and 5 are counterexamples

4. \( q_3(S_1, P_1) \)  
   False  
   \( \forall x \in S_1, x > 2 \rightarrow 0, 1, 2 \) are counterexamples

5. \( q_3(S_1, P_3) \)  
   False  
   \( \forall x \in S_1, x > 2 \land x > 3 \rightarrow 0, 1, 2, 3 \) are counterexamples.

6. \( q_2(S_1, P_3) \)  
   False  
   \( \forall x \in S_1, \neg(x > 2 \land x > 3) \rightarrow 4, 5 \) are counterexamples

7. \( q_1(S_1, P_4) \)  
   True  
   \( \exists x \in S_1, \neg(x > 3) \lor (x > 2) \rightarrow 0, 1, 2, 3, 4, 5 \) are examples.

8. \( q_0(S_1, P_4) \)  
   False  
   \( \exists x \in S_1, \neg((x > 3) \lor (x > 2)) \)  
   \( \equiv \neg(x > 3) \land \neg(x > 2) \rightarrow \text{no examples} \)
Question 2. [10 MARKS]

Part (a) [5 MARKS]

Consider the following symbolic statement:

\[ S_1 : \forall m \in \mathbb{N}, \forall n \in \mathbb{N}, \exists q \in \mathbb{N}, m \times n = q \]

1. Write the negation of the symbolic statement \( S_1 \), in such a way that the negation symbol \( \neg \) applies only to predicates such as \( m \times n = q \).

\[ \exists m \in \mathbb{N}, \exists n \in \mathbb{N}, \forall q \in \mathbb{N}, \neg (m \times n = q) \]

2. Which is true, statement \( S_1 \) or its negation? Briefly explain your reasoning.

\( S_1 \) is true.

For any \( m, n \in \mathbb{N} \), pick \( q = m \times n \), which must be a natural number.

Part (b) [5 MARKS]

Now consider the symbolic statement:

\[ S_2 : \exists q \in \mathbb{N}, \forall m \in \mathbb{N}, \forall n \in \mathbb{N}, m \times n = q \]

1. Write the negation of the symbolic statement \( S_2 \), in such a way that the negation symbol \( \neg \) applies only to predicates such as \( m \times n = q \).

\[ \forall q \in \mathbb{N}, \exists m \in \mathbb{N}, \exists n \in \mathbb{N}, \neg (m \times n = q) \]

2. Which is true, statement \( S_2 \) or its negation? Briefly explain your reasoning.

\( \neg S_2 \) is true.

If \( q > 0 \), pick \( m, n = 0 \). Otherwise, if \( q = 0 \) pick \( m = n = 1 \). Then \( m \times n \neq q \).
Question 3.  [6 marks]

Come up with an example of sets $D$, $P$, and $Q$ so that one of statements $S3, S4$ is true, and the other is false. Explain which is true, which false, and why.

Interpret $P(x)$ as $x \in P$, $Q(x)$ as $x \in Q$, and $D(x)$ as $x \in D$.

$S3$: $\exists x \in D, (P(x) \land \neg Q(x)) \lor (\neg P(x) \land Q(x))$

$S4$: $\exists x \in D, \neg P(x) \lor \neg Q(x)$

$$D = \{1, 2, 3\}$$
$$P = \{1, 3\}$$
$$Q = \{1, 3\}$$

Diagnosis: $D$ is a superset of both $P$ and $Q$. Therefore, $S4$ is not required, as it will always be false.

$S3$ is false. However, $S4$ is true.

There is no element either in the part of $P$ outside of $Q$, or the part of $Q$ outside $P$, so $S3$ is false. However, the element 2 is outside $P$, (also outside $Q$), so $S4$ is true.
This page is left (nearly) blank to accommodate work that wouldn’t fit elsewhere.

# 1: _____/22
# 2: _____/10
# 3: _____/ 6

TOTAL: _____/38