Do not turn this page until you have received the signal to start.
(In the meantime, please fill out the identification section above,
and read the instructions below.)
QUESTION 1. [22 marks]

Consider the following python functions and definitions, where \( L \) is a list and \( P \) is a boolean function. Note: feel free to ask about how python works, since you are being tested on logic, not programming.

\[
def q1(L, P) : \text{return False in } \{P(x) \text{ for } x \text{ in } L\}
def q2(L, P) : \text{return False not in } \{P(x) \text{ for } x \text{ in } L\}
def q3(L, P) : \text{return True in } \{P(x) \text{ for } x \text{ in } L\}
def q4(L, P) : \text{return True not in } \{P(x) \text{ for } x \text{ in } L\}
\]

\( L_1 = [0, 1, 2, 3, 4, 5] \)

\[
def P1(x) : \text{return } x > 2
def P2(x) : \text{return } x > 3
def P3(x) : \text{return } (\neg P2(x)) \text{ or } P1(x)
\]

\[
\begin{align*}
P2(x) & \Rightarrow P1(x) \\
x > 3 & \Rightarrow x > 2
\end{align*}
\]

PART (A) [6 marks]

Write the name of each function \( q1--q4 \) beside the comment(s) below that best describes the condition for which the function returns True. Indicate which are negations of each other.

1. \( \forall x \in L, P(x) \)

2. \( \exists x \in L, P(x) \)

3. \( \forall x \in L, \neg P(x) \)

4. \( \exists x \in L, \neg P(x) \)
PART (B) [16 marks]

Use your answer for the previous part to predict what the output is below. For each answer, briefly explain your thinking.

\[ p_1: x > 2 \]

1. \( q_1(L_1, P_1) \)  
   True - \( p_1(1) \) is False, so 
   \( \exists x \in L_1, \neg p_1(x) \)

2. \( q_2(L_1, P_1) \)  
   False - negation of \( q_1 \)

3. \( q_3(L_1, P_1) \)  
   True - \( p_1(3) \) is True, so 
   \( \exists x \in L_1, p_1(x) \)

4. \( q_4(L_1, P_1) \)  
   False - negation of \( q_3 \)

5. \( q_1(L_1, P_3) \)  
   False - negation of \( q_2 \)

6. \( q_2(L_1, P_3) \)  
   True - \( x > 3 \Rightarrow x > 2 \) for all \( x \in L_1 \)

7. \( q_3(L_1, P_3) \)  
   True - \( p_3(4) \) is True, so 
   \( \exists x \in L_1, p_3(x) \) is True

8. \( q_4(L_1, P_3) \)  
   False - negation of \( q_3 \)
QUESTION 2.  [10 marks]

PART (A)  [5 marks]
Consider the following symbolic statement:

\[ S_1 : \forall \varepsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, \forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > \varepsilon \]

1. Write the negation of the symbolic statement \( S_1 \), in such a way that the negation symbol \( \neg \) applies only to predicates such as \( x > \delta \) or \( x^3 > \varepsilon \).

\[ \exists \varepsilon \in \mathbb{R}^+, \forall \delta \in \mathbb{R}^+, \exists x \in \mathbb{R}, x > \delta \land \neg (x^3 > \varepsilon) \]

2. Which is true, statement \( S_1 \) or its negation? Briefly explain your reasoning.

\( S_1 \) is true. Suppose your enemy chooses \( \varepsilon > 0 \). You then choose \( \delta = \sqrt[3]{3\varepsilon} > 0 \). Then \( \forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > \delta^3 = (\sqrt[3]{3\varepsilon})^3 = \varepsilon \)

PART (B)  [5 marks]
Now the consider the symbolic statement:

\[ S_2 : \exists \delta \in \mathbb{R}^+, \forall \varepsilon \in \mathbb{R}^+, \forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > \varepsilon \]

1. Write the negation of the symbolic statement \( S_2 \), in such a way that the negation symbol \( \neg \) applies only to predicates such as \( x > \delta \) or \( x^3 > \varepsilon \).

\[ \forall \delta \in \mathbb{R}^+, \exists \varepsilon \in \mathbb{R}^+, \exists x \in \mathbb{R}, x > \delta \land \neg (x^3 > \varepsilon) \]

2. Which is true, statement \( S_2 \) or its negation? Briefly explain your reasoning.

\( S_2 \) is true. Suppose your enemy chooses \( \delta \). You then choose \( \varepsilon = 8\delta^3 \) and \( x = 2\delta \). Then \( x = 2\delta > \delta \) and \( x^3 = 8\delta^3 \leq \varepsilon \)
QUESTION 3.  [6 marks]

Suppose $F$ is the set of functions, $D(f)$ means “$f$ is differentiable,” and $C(f)$ means “$f$ is continuous.” Consider the following statement:

$S3$ : “Every function is not differentiable unless it is continuous.”

Write the contrapositive, converse, and the negation of $S3$ symbolically. Note: in this course we translate “unless” as “if not”.

Contrapositive: $\forall f \in F, D(f) \Rightarrow C(f)$

Converse: $\forall f \in F, \neg D(f) \Rightarrow \neg C(f)$

Negation: $\exists f \in F, \neg C(f) \land D(f)$
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# 1: _____/22

# 2: _____/10

# 3: _____/ 6

TOTAL: _____/38