Wacky Wed—what about tutorials, stay tuned?
A2—getting closer...
We’ve proved: \( P(n) : 2^n \geq 2n \)

Use this to prove that \( 2^n \not\in \mathcal{O}(n) \)

\[ \forall c \in \mathbb{R}^+, \forall B \in \mathbb{N}, \exists n \in \mathbb{N}, \quad n \geq B \land 2^n > cn \]

Assume \( c \in \mathbb{R}^+ \), assume \( B \in \mathbb{N} \)

Choose \( n = 2 \left( \lceil \log c \rceil + B + 1 \right) \)

Then \( n \in \mathbb{N} \)

\# \( \lceil \log c \rceil \in \mathbb{N}, B, 1, c \in \mathbb{N}, \mathbb{N} \) closed under +

Then \( n \geq B \)

Then \( 2^n = 2^{n/2} 2^{n/2} \)

\[ \geq 2^{n/2} \cdot n \]

\[ > cn \]

\[ \lim_{x \to \infty} \frac{2^x}{x} \]

\[ = \log x \]

\[ 2 \log x = x \]
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bounded below

Notice that the definition of big-Omega differs in just one character from big-Oh:

\[ \Omega(g) = \{ f : \mathbb{N} \to \mathbb{R}_{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \geq cg(n) \} \]

The rôle of \( B \) is, as with big-Oh, to act as a breakpoint, so comparisons don't have to start at the origin.

The rôle of \( c \) is to scale \( g \) down below \( f \).

If you’re proving \( f \in \Omega(g) \), you get to choose \( c \) and \( B \) to suit your proof.
It often happens that functions are bounded above and below by the same function. In other words, $f \in \mathcal{O}(G) \land f \in \Omega(g)$. We combine these two concepts into $f \in \Theta(g)$.

$$\Theta(g) = \{ f : \mathbb{N} \mapsto \mathbb{R}_{\geq 0} \mid \exists c_1 \in \mathbb{R}^+, \exists c_2 \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow c_1 g(n) \leq f(n) \leq c_2 g(n) \}$$

You might want to draw pictures, and conjecture about values of $c_1, c_2, B$ for $f = 5n^2 + 15$ and $g = n^2$. 
How do you deal with a general statement about two functions:

\[(f \in O(g) \wedge g \in O(h)) \Rightarrow f \in O(h)\]

Assume \(f, g, h \in f\),

Assume \(f \in O(g) \wedge g \in O(h) \neq \text{antecedent}\)

Then \(\exists c \in \mathbb{R}^+, \exists B, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq c \cdot g(n)\) \((\star)\)

Choose \(c_1 \in \mathbb{R}^+, B_1 \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B_1 \Rightarrow f(n) \leq c_1 \cdot g(n)\) \((\star \star)\)

Then \(\exists c_2 \in \mathbb{R}^+, \exists B_2 \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B_2 \Rightarrow g(n) \leq c_2 \cdot h(n)\) \((\star \star \star)\)

Choose \(c' = c_1 \cdot c_2\). Then \(c' \in \mathbb{R}^+\)

Choose \(B' = \max(B_1, B_2)\)

Then \(\forall n \in \mathbb{N}, n \geq B' \Rightarrow f(n) \leq c_1 \cdot g(n) \leq c_1 \cdot c_2 \cdot h(n)\) \((\star \star \star \star)\)

Then \(f \in O(h)\)
How about: $g \in \Omega(f) \iff f \in \mathcal{O}(g)$
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