pinning down intuition

We know, or have heard, that polynomials of the same degree grow at “roughly” the same speed.
We want to make this “roughly” explicit.
Here’s how we define $O(n^2)$, functions that eventually grow no more quickly than $n^2$

$$O(n^2) = \{ f : \mathbb{N} \mapsto \mathbb{R}^\geq_0 | \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cn^2 \}$$

The definition says that there’s a multiplier, $c$, such that if you go far enough to the right, $B$, the graph of $f$ is bounded above by the graph of $cn^2$. 
Prove \( 3n^2 + 2n \in O(n^2) \) \\
\[ n \geq B \Rightarrow 3n^2 + 2n \leq cn^2 \]

\[ 3n^2 + 2n : \mathbb{N} \rightarrow \mathbb{R}^+ \]

Let \( c = 5 \). Then \( c \in \mathbb{R}^+ \) \\
Let \( B = 0 \). Then \( B \in \mathbb{N} \) \\
Assume \( n \in \mathbb{N} \) \# generic choice

Assume \( n \geq B \) \# antecedent

Then \( 3n^2 + 2n \leq 3n^2 + 2n \cdot n \) \# \( n=0 \) then \( 2n^2 = 2n \)

\[ = 5n^2 \]

\[ = cn^2 \] \# \( c = 5 \)

Then \( n \geq B \Rightarrow 3n^2 + 2n \leq cn^2 \) \# introduced \( \Rightarrow \)

Then \( \forall n \in \mathbb{N}, n \geq B \Rightarrow 3n^2 + 2n \leq cn^2 \) \# introduced \( \forall \)

Then \( \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow 3n^2 + 2n \leq cn^2 \) \# introduced \( \exists \) twice

Conclude \( 3n^2 + 2n \in O(n^2) \) \# matches definition
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in general, $O(g)$:

$$O(g) = \{ f : \mathbb{N} \rightarrow \mathbb{R}^+ \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cg(n) \}$$

Prove: $7n^6 - 5n^4 + 2n^3 \in O(6n^8 - 4n^5 + n^2)$

Let $c = \text{_____}$. Then $c \in \mathbb{R}^+$.

Let $B = \text{_____}$. Then $B \in \mathbb{N}$.

Assume $n \in \mathbb{N}$ # generic choice

Assume $n \geq B$ # antecedent

Then $7n^6 - 5n^4 + 2n^3 \leq 7n^6 + 2n^3$ # added $5n^4 \geq 0$

$\leq 7n^6 + 2n^6$ # mult by $n^3$

$= 9n^6$ # won't decrease

$\leq 9n^8$ # mult by $n^2$ won't decrease

$= c2n^8$ # $c = 9/2$

$= c(6n^8 - 4n^8)$

$\leq c(6n^8 - 4n^5) \iff 4n^8 \geq 4n^5 \forall n$

$\leq c(6n^8 - 4n^5 + n^2)$ # added $n^2 \geq 0$

Then $n \geq B \implies 7n^6 - 5n^4 + 2n^3 \leq c(6n^8 - 4n^5 + n^2)$ # intro
\[
\forall n \in \mathbb{N}, n \geq B \quad \Rightarrow \quad 7n^6 - 5n^4 + 2n^3 \leq c(6n^8 - 4n + n^2)
\]

Then \( \exists c \in \mathbb{R}^+ \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow 7n^6 - 5n^4 + 2n^3 \leq c(6n^8 - 4n + n^2) \)

\# intro \( \exists \) twice

conclude \( 7n^6 - 5n^4 + 2n^3 \in O(6n^8 - 4n + n^2) \) \# matches defn.
how to prove $n^4 \not\in \mathcal{O}(3n^2)$

begin by stating the negation, then prove that
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