CSC148 winter 2014
recursive structures
week 6

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Outline
recursion, natural and otherwise
terminology

- set of nodes (possibly with values or labels), with directed edges between some pairs of nodes

- One node is distinguished as root

- Each non-root node has exactly one parent.

- A path is a sequence of nodes $n_1, n_2, \ldots, n_k$, where there is an edge from $n_i$ to $n_{i+1}$. The length of a path is the number of edges in it

- There is a unique path from the root to each node. In the case of the root itself this is just $n_1$, if the root is node $n_1$.

- There are no cycles — no paths that form loops.
more terminology

- **leaf**: node with no children

- **internal node**: node with one or more children

- **subtree**: tree formed by any tree node together with its descendants and the edges leading to them.

- **height**: Maximum path length in a tree. A node also defines a height, which is the maximum path length of the tree rooted at that node

- **arity, branching factor**: maximum number of children for any node.
pre-order traversal

Visit root, then pre-order left subtree, then pre-order right subtree
A TreeList is either None or a Python list with 3 elements, where
--- element 0 is a value
--- element 1 is a TreeList
--- element 2 is a TreeList

def preorder(tl: 'TreeList') -> list:
    """
    Return list of values in tl in preorder
    """
    >>> T = [5, [4, None, None], [3, [2, None, None], [1, None, None]]]
    >>> preorder(T)
    [5, 4, 3, 2, 1]
    """
in-order traversal

Visit in-order left subtree, then root, then in-order right subtree
exercise: code for inorder traversal

A TreeList is either None or a Python list with 3 elements, where
--- element 0 is a value
--- element 1 is a TreeList
--- element 2 is a TreeList

```
def inorder(tl: 'TreeList') -> list:
    """
    Return list of values in tl in order
    """
    >>> T = [5, [4, None, None], [3, [2, None, None], [1, None, None]]
    >>> inorder(T)
    [4, 5, 2, 3, 1]
    """
```
post-order traversal

Visit post-order left subtree, then post-order right subtree, then root
exercise: code for postorder traversal

A TreeList is either None or a Python list with 3 elements, where
--- element 0 is a value
--- element 1 is a TreeList
--- element 2 is a TreeList

```python
def postorder(tl: 'TreeList') -> list:
    """
    Return list of values in tl in postorder
    """
    >>> T = [5, [4, None, None], [3, [2, None, None], [1, None, None]]]
    >>> postorder(T)
    [4, 2, 1, 3, 5]
    """
```
general tree implementation

Python list class has way more methods and attributes than needed. Let’s specialize on Tree ADT.

class Tree:
    def __init__(self: 'Tree',
                 value: object = None, children: list = None):
        """Create a node with value and any number of children""

        self.value = value
        if not children:
            self.children = []
        else:
            self.children = children[:].copy() # quick-n-dirty copy of list

    def __contains__(self: 'Tree', value: object) -> bool:
        """True if Tree has a node with value""
        return (self.value == value or
                any([t.__contains__(value) for t in self.children]))
add a string representation

def __repr__(self: 'Tree') -> str:
    """Return representation of Tree as a string"""
def count(t: Tree) -> int:
    """How many nodes in this Tree?

    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
    >>> tn3 = Tree(3, [Tree(6), Tree(7)])
    >>> tn1 = Tree(1, [tn2, tn3])
    >>> count(tn1)
    9
    """
def height(t: Tree) -> int:
    """Return length of longest path of t"

>>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
>>> tn3 = Tree(3, [Tree(6), Tree(7)])
>>> tn1 = Tree(1, [tn2, tn3])
>>> height(tn1)
2
"""

# 1 more edge than the maximum height of a child, except
# what happens if there are no children?
how many leaves?

```python
def leaf_count(t: Tree) -> int:
    """Return number of leaves in t"

>>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
>>> tn3 = Tree(3, [Tree(6), Tree(7)])
>>> tn1 = Tree(1, [tn2, tn3])
>>> leaf_count(tn1)
6
"""
```
def arity(t: Tree) -> int:
    """Maximum branching factor of tree T"

    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
    >>> tn3 = Tree(3, [Tree(6), Tree(7)])
    >>> tn1 = Tree(1, [tn2, tn3])
    >>> arity(tn1)
    4
    """