Note that in this exercise we perform GAC as preprocessing in advance of any search, *as well as* during search. Constraint propagation (GAC, FC, etc.) is often performed in advance of initiating search as a preprocessing step to prune variable domains. Once no more propagation can be done, search (with GAC, FC, etc. as illustrated in our algorithms) is commenced. The example that follows illustrates the additional use of GAC as a preprocessing step in advance of commencing search with GAC.
GAC Example

(a) \( \text{Dom}[X] = \{1, 2, 3, 4\} \)
(b) \( \text{Dom}[Y] = \{1, 2, 3, 4\} \)
(c) \( \text{Dom}[Z] = \{1, 2, 3, 4\} \)
(d) \( \text{Dom}[W] = \{1, 2, 3, 4, 5\} \)

And 3 constraints:

(a) \( C_1(X, Y, Z) \) which is satisfied only when \( X = Y + Z \)
(b) \( C_2(X, W) \) which is satisfied only when \( W > X \)
(c) \( C_3(X, Y, Z, W) \) which is satisfied only when \( W = X + Z + Y \)

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.
GAC Example

(a) \( \text{Dom}[X] = \{1, 2, 3, 4\} \)
(b) \( \text{Dom}[Y] = \{1, 2, 3, 4\} \)
(c) \( \text{Dom}[Z] = \{1, 2, 3, 4\} \)
(d) \( \text{Dom}[W] = \{1, 2, 3, 4, 5\} \)

And 3 constraints:

(a) \( C_1(X, Y, Z) \) which is satisfied only when \( X = Y + Z \)
(b) \( C_2(X, W) \) which is satisfied only when \( W > X \)
(c) \( C_3(X, Y, Z, W) \) which is satisfied only when \( W = X + Z + Y \)

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- All constraints put on GAC queue.
- Process \( C_3 \) first.
  - \( X = 1 \) (\( X=1, Y=1, Z=1, W=3 \))
  - \( X = 2 \) (\( X=2, Y=1, Z=1, W=4 \))
  - \( X = 3 \) (\( X=3, Y=1, Z=1, W=5 \))
  - \( X = 4 \) – Inconsistent.
  - Dom(\( X \)) = \( \{1, 2, 3\} \)
    - similarly
  - Dom(\( Y \)) = \( \{1, 2, 3\} \)
  - Dom(\( Z \)) = \( \{1, 2, 3\} \)
  - \( W = 1 \) – inconsistent
  - \( W = 2 \) – inconsistent
  - \( W = 3 \) – same support as \( X=1 \)
  - \( W = 4 \) – same support as \( X = 2 \)
  - \( W = 5 \) – same support as \( X = 3 \)
  - Dom(\( W \)) = \( \{3, 4, 5\} \)

All domains pruned, but all other constraints already on GAC queue.
GAC Example

(a) $\text{Dom}[X] = \{1, 2, 3, 4\}$  
(b) $\text{Dom}[Y] = \{1, 2, 3, 4\}$  
(c) $\text{Dom}[Z] = \{1, 2, 3, 4\}$  
(d) $\text{Dom}[W] = \{1, 2, 3, 4, 5\}$

And 3 constraints:

(a) $C_1(X, Y, Z)$ which is satisfied only when $X = Y + Z$
(b) $C_2(X, W)$ which is satisfied only when $W > X$
(c) $C_3(X, Y, Z, W)$ which is satisfied only when $W = X + Z + Y$

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- Process $C_2$ next
  - Currently  
    - $\text{Dom}(X) = \{1, 2, 3\}$  
    - $\text{Dom}(W) = \{3, 4, 5\}$
  - $X = 1$ (X=1, W=3)  
  - $X = 2$ (X=2, W=3)  
  - $X = 3$ (X=3, W=4)

  $W=5$ (X=1, W=5)  
  No domains pruned. Nothing added to GAC Queue

  $W=3, W=4$ found supports already
GAC Example

(a) \( \text{Dom}[X] = \{1, 2, 3, 4\} \)
(b) \( \text{Dom}[Y] = \{1, 2, 3, 4\} \)
(c) \( \text{Dom}[Z] = \{1, 2, 3, 4\} \)
(d) \( \text{Dom}[W] = \{1, 2, 3, 4, 5\} \)

And 3 constraints:

(a) \( C_1(X, Y, Z) \) which is satisfied only when \( X = Y + Z \)
(b) \( C_2(X, W) \) which is satisfied only when \( W > X \)
(c) \( C_3(X, Y, Z, W) \) which is satisfied only when \( W = X + Z + Y \)

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- **Process \( C_1 \) next**

  At this stage
  \[
  \text{Dom}(X) = \text{Dom}(Y) = \text{Dom}(Z) = \{1, 2, 3\}
  \]

  \( X = 1 \) – inconsistent
  \( X = 2 \) – (\( X=2, Y=1, Z=1 \))
  \( X = 3 \) – (\( X=3, Y=1, Z=2 \))

  \( Y = 1 \) – same support as \( X=2 \)
  \( Y = 2 \) – (\( X=3, Y=2, Z=1 \))
  \( Y = 3 \) – inconsistent

  \( Z = 1 \) – same support as \( X=2 \)
  \( Z = 2 \) – same support as \( X=3 \)
  \( Z = 3 \) – inconsistent

  Updated domains
  \( X = \{2,3\} \)
  \( Y = \{1,2\} \)
  \( Z = \{1,2\} \)

  Put \( C_2 \) and \( C_3 \) back onto GAC queue
GAC Example

(a) \( \text{Dom}[X] = \{1, 2, 3, 4\} \)
(b) \( \text{Dom}[Y] = \{1, 2, 3, 4\} \)
(c) \( \text{Dom}[Z] = \{1, 2, 3, 4\} \)
(d) \( \text{Dom}[W] = \{1, 2, 3, 4, 5\} \)

And 3 constraints:

(a) \( C_1(X, Y, Z) \) which is satisfied only when \( X = Y + Z \)
(b) \( C_2(X, W) \) which is satisfied only when \( W > X \)
(c) \( C_3(X, Y, Z, W) \) which is satisfied only when \( W = X + Z + Y \)

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- Process \( C_3 \) next current domains:
  - \( \text{Dom}(X) = \{2, 3\} \)
  - \( \text{Dom}(Y) = \{1, 2\} \)
  - \( \text{Dom}(Z) = \{1, 2\} \)
  - \( \text{Dom}(W) = \{3, 4, 5\} \)
  - \( X = 2 - \{X=2, W=4, Y=1, Z=1\} \)
  - \( X = 3 - \{X=3, W=5, Y=1, Z=1\} \)
  - \( Y = 1 - \text{found support} \)
  - \( Y = 2 - \{X=2, W=5, Y=2, Z=1\} \)
  - \( Z = 1 - \text{found support} \)
  - \( Z = 2 - \{X=2, W=5, Y=1, Z=2\} \)
  - \( W = 3 \text{ inconsistent} \)
  - \( W = 4 - \text{found support} \)
  - \( W = 5 - \text{found support} \)

Pruned domains
- \( W = \{4, 5\} \)

\( C_2 \) already on GAC queue
GAC Example

(a) \(\text{Dom}[X] = \{1, 2, 3, 4\}\)
(b) \(\text{Dom}[Y] = \{1, 2, 3, 4\}\)
(c) \(\text{Dom}[Z] = \{1, 2, 3, 4\}\)
(d) \(\text{Dom}[W] = \{1, 2, 3, 4, 5\}\)

And 3 constraints:

(a) \(C_1(X, Y, Z)\) which is satisfied only when \(X = Y + Z\)
(b) \(C_2(X, W)\) which is satisfied only when \(W > X\)
(c) \(C_3(X, Y, Z, W)\) which is satisfied only when \(W = X + Z + Y\)

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- Process \(C_2\) next current domains:
  \(\text{Dom}(X) = \{2, 3\}\)
  \(\text{Dom}(W) = \{4, 5\}\)
  \(X = 2 \rightarrow \{X=2, W=4\}\)
  \(X = 3 \rightarrow \{X=3, W=4\}\)
  \(W = 4 \rightarrow \text{found support}\)
  \(W = 5 \rightarrow \{X=3, W=5\}\)

  No Domains pruned.
  Nothing added to queue
  Queue Empty

  GAC finished.

  GAC domains:
  \(X = \{2,3\}\)
  \(Z = \{1, 2\}\)
  \(Y = \{1, 2\}\)
  \(W = \{4,5\}\)
GAC Example

(a) $Dom[X] = \{1, 2, 3, 4\}$
(b) $Dom[Y] = \{1, 2, 3, 4\}$
(c) $Dom[Z] = \{1, 2, 3, 4\}$
(d) $Dom[W] = \{1, 2, 3, 4, 5\}$

And 3 constraints:

(a) $C_1(X, Y, Z)$ which is satisfied only when $X = Y + Z$
(b) $C_2(X, W)$ which is satisfied only when $W > X$
(c) $C_3(X, Y, Z, W)$ which is satisfied only when $W = X + Z + Y$

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- **Note GAC enforce does not find a solution**
  To find a solution we must use do search while enforcing GAC.

- **Branch on X.**
  - $X = 2$
  - $GAC(C_1) \Rightarrow Y = 1, Z=1$
  - $GAC(C_2) \Rightarrow$ no changes
  - $GAC(C_3) \Rightarrow W = 4$
  - This is a solution.

- **Branch on X = 3**
  - $GAC(C_1) \Rightarrow$ no changes
  - $GAC(C_2) \Rightarrow$ no changes
  - $GAC(C_3) \Rightarrow$ Prune $W=4$
    - Prune $Y = 2$
    - Prune $Z = 2$

  **Current Domains**
  - $X={3}$, $Y={1}$, $Z={1}$, $W={5}$

  - $GAC(C_1) \Rightarrow$ Prune $Y={1}$ DWO

  **NOTE No solution with X=3 but X=3 not pruned by GAC enforce.**
Example

- C1(V1, V2, V3)
  - V1 | V2 | V3
  - A  | B  | C
  - B  | A  | C
  - A  | A  | B

- C2(V1, V3, V4, V5)
  - V1 | V3  | V4 | V5
  - A  | A   | A  | A
  - A  | B   | C  | B
  - B  | C   | B  | B
  - C  | A   | B  | C
  - C  | B   | A  | B

- C3(V2, V3, V5)
  - V2 | V3  | V5
  - A  | A   | A
  - A  | B   | C
  - B  | C   | B
  - C  | A   | B
  - C  | B   | A

- Dom[V1]...Dom[V5] = {a, b, c}
### Example

#### C1(V1,V2,V3)

<table>
<thead>
<tr>
<th>V1</th>
<th>V2</th>
<th>V3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

- V1=C: no support
- V2=C: no support
- V3=A: no support

#### C2(V1,V3,V4,V5)

<table>
<thead>
<tr>
<th>V1</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
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<tr>
<td>A</td>
<td>B</td>
<td>C</td>
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<tr>
<td>C</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

#### C3(V2,V3,V5)

<table>
<thead>
<tr>
<th>V2</th>
<th>V3</th>
<th>V5</th>
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</thead>
<tbody>
<tr>
<td>A</td>
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<tr>
<td>A</td>
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<tr>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>A</td>
</tr>
</tbody>
</table>

- V1={a,b}
- V2={a,b}
- V3={b,c}
Example

- **C1(V1,V2,V3)**
  
<table>
<thead>
<tr>
<th>V1</th>
<th>V2</th>
<th>V3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

- **C2(V1,V3,V4,V5)**
  
<table>
<thead>
<tr>
<th>V1</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
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<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
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<tr>
<td>A</td>
<td>B</td>
<td>C</td>
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<tr>
<td>C</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

- **C3(V2,V3,V5)**
  
<table>
<thead>
<tr>
<th>V2</th>
<th>V3</th>
<th>V5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>A</td>
</tr>
</tbody>
</table>

- V1=C: no support
- V2=C: no support
- V3=A: no support
- V1=\{a,b\}
- V2=\{a,b\}
- V3=\{b,c\}
**Example**

- **C1(V1,V2,V3)**
  - | V1 | V2 | V3 |
  - | A  | B  | C  |
  - | B  | A  | C  |
  - | A  | A  | B  |

- **C2(V1,V3,V4,V5)**
  - | V1 | V3 | V4 | V5 |
  - | A  | A  | A  | A  |
  - | A  | B  | C  | B  |
  - | B  | C  | B  | B  |
  - | C  | A  | B  | C  |
  - | C  | B  | A  | B  |

- **C3(V2,V3,V5)**
  - | V2 | V3 | V5 |
  - | A  | A  | A  |
  - | A  | B  | C  |
  - | B  | C  | B  |
  - | C  | A  | B  |
  - | C  | B  | A  |

- **V1=C**: no support
- **V2=C**: no support
- **V3=A**: no support

- **V1={a,b}**
- **V2={a,b}**
- **V3={b,c}**

- **V4=A**: no support
- **V5=A**: no support
- **V5=C**: no support

- **V4={C,B}**
- **V5={B}**
### Example

- **C1(V1,V2,V3)**

<table>
<thead>
<tr>
<th>V1</th>
<th>V2</th>
<th>V3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

- **V1=C: no support**
- **V2=C: no support**
- **V3=A: no support**

- **V1={}a,b{}**
- **V2={}a,b{}**
- **V3={}b,c{}**

- **C2(V1,V3,V4,V5)**

<table>
<thead>
<tr>
<th>V1</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

- **V4=A: no support**
- **V5=A: no support**
- **V5=C: no support**

- **V4={}C,B{}**
- **V5={}B{}**

- **C2(V2,V3,V5)**

<table>
<thead>
<tr>
<th>V2</th>
<th>V3</th>
<th>V5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>A</td>
</tr>
</tbody>
</table>
**Example**

- **C1(V1,V2,V3)**
  - V1 | V2 | V3
  - A  | B  | C
  - B  | A  | C
  - A  | A  | B

  - V1=C: no support
  - V2=C: no support
  - V3=A: no support

- **C2(V1,V3,V4,V5)**
  - V1 | V3 | V4 | V5
  - A  | A  | A  | A
  - A  | B  | C  | B
  - B  | C  | B  | B
  - C  | A  | B  | C
  - C  | B  | A  | B

  - V4=A: no support
  - V5=A: no support
  - V5=C: no support

- **C2(V2,V3,V5)**
  - V2 | V3 | V5
  - A  | A  | A
  - A  | B  | C
  - B  | C  | B
  - C  | A  | B
  - C  | B  | A

  - V2=A: no support
  - V3=B: no support

- **C2(V2,V3,V5)**
  - V2 | V3 | V5
  - B  | C  | B
  - C  | A  | B
  - C  | B  | A

  - V2={B}
  - V3={C}
### Example

- **C1(V1,V2,V3)**

<table>
<thead>
<tr>
<th>V1</th>
<th>V2</th>
<th>V3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

- **C2(V1,V3,V4,V5)**

<table>
<thead>
<tr>
<th>V1</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
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<td>C</td>
<td>A</td>
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<td>C</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

- **C2(V2,V3,V5)**

<table>
<thead>
<tr>
<th>V2</th>
<th>V3</th>
<th>V5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

- V1=B has no support
- V1={A}
- V4={C,B}
- V5={B}
- V2={B}
- V3={C}
### Example

<table>
<thead>
<tr>
<th></th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

- **C1(V1,V2,V3)**

<table>
<thead>
<tr>
<th>V1</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

- **C2(V1,V3,V4,V5)**

<table>
<thead>
<tr>
<th>V1</th>
<th>V2</th>
<th>V3</th>
<th>V5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
</tbody>
</table>

- **C2(V2,V3,V5)**

<table>
<thead>
<tr>
<th>V2</th>
<th>V3</th>
<th>V5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>A</td>
</tr>
</tbody>
</table>

- V1=B has no support
- V1={A}
- V4=B has no support
- V4={B}
- V5={B}
- V3=C has no support
- V3={} DWO
- V2={B}
- V3={C}