Exercise Worksheet (Deadlock avoidance)

1. Consider that 3 wizards from Hogwarts: Ron (R), Hermione (H), and Dumbledore (D), are trying to create complex spells that require 2 types of potions: Confusing Concoction (CC), and Felix Felicis (FF).

Each wizard needs an exact total number of potions of each type, in order to complete their spell, so they make gradually more requests for extra potions, depending on what they need. Once a spell is complete, the corresponding number of potions of the types used for the spell magically replenish and become available again for other wizards to request.

At this point, the wizards have already been given a number of potions of each type:
R (2, 1)  H (1, 2)  D (0, 0)
So, for example, Ron has successfully obtained 2 CC potions, and 1 FF potion.

The number of potions which are still available (not assigned to any of the wizards yet) are 1 of each type.

The total number of potions necessary for each of the wizards to complete their spell are:
R (3, 3)  H (2, 2)  D (1, 2)

Answer the questions below, explaining your rationale, then compare with your neighbour:

a) Is our potion allocation system currently able to satisfy the wizards’ needs, given some chain of requests? In other words, is there at least one chain of potion requests, where everyone can get to make their spells successfully? Keep in mind that potions become available again once a spell is complete!

b) If in this current state, Ron requests one CC potion, should this request be granted?
2. Consider that now Harry Potter (P) joins in and that everyone has to complete other more complex spells. Each wizard now needs to create spells using 3 types of potions: CC, FF, and PP (Polyjuice Potion).

After a series of potion requests being granted, the wizards have now already been granted a number of potions of each type:
R (0, 2, 1)       H (1, 3, 2)       D (3, 1, 1)       P (1, 4, 3)
For example, Harry Potter (P) has already been given 1 CC potion, 4 FF potions, and 3 PP potions.

The number of potions available (that can still be requested) is 2 of each type.

The total number of potions necessary for each wizard to complete their spell are:
R (7, 4, 2)       H (3, 5, 3)       D (6, 7, 8)       P (4, 9, 6)

Similarly to the previous exercise, a wizard cannot complete their spell without having all the right potions in the right amounts, but once a spell is completed, the corresponding number of potions used for that spell, magically get replenished and become available to be requested by other wizards.

Answer the questions below, explaining your rationale, then compare with your neighbour:

a) Is our potion allocation system currently able to satisfy the wizards’ needs, given some chain of requests? In other words, is there at least one chain of potion requests, where everyone can get to make their spells successfully? Keep in mind that potions become available again once a spell is complete!

b) If in this current state, Ron requests one PP potion, should this request be granted?