Design Theory for Relational Databases

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Originally based on slides by Jeff Ullman
There are always many different schemas for a given set of data.

E.g., you could combine or divide tables.

How do you pick a schema?
Which is better?
What does “better” mean?

Fortunately, there are some principles to guide us.
Database Design Theory

◆ It allows us to improve a schema systematically.
◆ General idea:
  ◦ Express constraints on the relationships between attributes
  ◦ Use these to decompose the relations
◆ Ultimately, get a schema that is in a “normal form” that guarantees good properties.
◆ “Normal” in the sense of conforming to a standard.
◆ The process of converting a schema to a normal form is called normalization.
Part I:
Functional Dependency Theory
A poorly designed table

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◆ In any domain, there are relationships between attribute values.
◆ Perhaps:
  ◆ Every part has 1 manufacturer
  ◆ Every manufacture has 1 address
  ◆ Every seller has 1 address
◆ If so, this table will have redundant data.
Principle: Avoid redundancy

Redundant data can lead to anomalies.

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- **Update anomaly**: if Hammers ‘R Us moves and we update only one tuple, the data is inconsistent.
- **Deletion anomaly**: If ABC stops selling part 8624 and Lee Valley makes only that one part, we lose track of its address.
Definition of FD

◆ Suppose R is a relation, and X and Y are subsets of the attributes of R.

◆ $X \rightarrow Y$ asserts that:
  ▪ If two tuples agree on all the attributes in set $X$, they must also agree on all the attributes in set $Y$.

◆ We say that “$X \rightarrow Y$ holds in R”, or “$X$ functionally determines $Y$.”

◆ An FD constrains what can go in a relation.
More formally...

A → B means:

∀ tuples t₁, t₂,
(t₁[A] = t₂[A]) ⇒ (t₁[B] = t₂[B])

Or equivalently:

¬ ∃ tuples t₁, t₂ such that
(t₁[A] = t₂[A]) ∧ (t₁[B] ≠ t₂[B])
Generalization to multiple attributes

\[ A_1 A_2 \ldots A_m \to B_1 B_2 \ldots B_n \] means:

\[ \forall \text{tuples } t_1, t_2, \]
\[ (t_1[A_1] = t_2[A_1] \land \ldots \land t_1[A_m] = t_2[A_m]) \implies \]
\[ (t_1[B_1] = t_2[B_1] \land \ldots \land t_1[B_n] = t_2[B_n]) \]

Or equivalently:

\[ \neg \exists \text{tuples } t_1, t_2 \text{ such that} \]
\[ (t_1[A_1] = t_2[A_1] \land \ldots \land t_1[A_m] = t_2[A_m]) \land \]
\[ \neg (t_1[B_1] = t_2[B_1] \land \ldots \land t_1[B_n] = t_2[B_n]) \]
Why “functional dependency”?

◆ “dependency” because the value of \( Y \) depends on the value of \( X \).

◆ “functional” because there is a mathematical function that takes a value for \( X \) and gives a *unique* value for \( Y \).

◆ (It’s not a typical function; just a lookup.)
Equivalent sets of FDs

◆ When we write a set of FDs, we mean that all of them hold.
◆ We can very often rewrite sets of FDs in equivalent ways.
◆ When we say $S_1$ is equivalent to $S_2$ we mean that:
  ◦ $S_1$ holds in a relation iff $S_2$ does.
Splitting rules for FDs

◆ Can we split the RHS of an FD and get multiple, equivalent FDs?

◆ Can we split the LHS of an FD and get multiple, equivalent FDs?
Coincidence or FD?

- An FD is an assertion about every instance of the relation.
- You can’t know it holds just by looking at one instance.
- You must use knowledge of the domain to determine whether an FD holds.
FDs are closely related to keys

- Suppose K is a set of attributes for relation R.
- Our old definition of superkey:
  a set of attributes for which no two rows can have the same values.
- A claim about FDs:
  \( K \) is a superkey for \( R \) iff
  \( K \) functionally determines all of \( R \).
FDs are a generalization of keys

- **key:**
  \[ X \rightarrow R \]
  Every attribute

- **Functional dependency:**
  \[ X \rightarrow Y \]
  Not necessarily every attribute

- An FD can be more subtle.
Inferring FDs

◆ Given a set of FDs, we can often infer further FDs.
◆ This will be handy when we apply FDs to the problem of database design.
◆ Big task: given a set of FDs, infer every other FD that must also hold.
◆ Simpler task: given a set of FDs, check whether a given FD must also hold.
Examples

◆ If $A \rightarrow B$ and $B \rightarrow C$ hold, must $A \rightarrow C$ hold?

◆ If $A \rightarrow H$, $C \rightarrow F$, and $F G \rightarrow A D$ hold, must $F A \rightarrow D$ hold?
  must $CG \rightarrow FH$ hold?

◆ If $H \rightarrow GD$, $HD \rightarrow CE$, and $BD \rightarrow A$ hold, must $EH \rightarrow C$ hold?

◆ Aside: we are not generating new FDs, but testing a specific possible one.
Method 1: Prove an FD follows using first principles

◆ You can prove it by referring back to
  ◢ The FDs that you know hold, and
  ◢ The definition of functional dependency.
◆ But the Closure Test is easier.
Method 2: Prove an FD follows using the Closure Test

◆ Assume you know the values of the LHS attributes, and figure out everything else that is determined.
◆ If it includes the RHS attributes, then you know that LHS $\rightarrow$ RHS
◆ This is called the closure test.
Y is a set of attributes, S is a set of FDs.
Return the closure of Y under S.

Attribute_closure(Y, S):
Initialize $Y^+$ to Y
Repeat until no more changes occur:
  If there is an FD LHS $\rightarrow$ RHS in S such that LHS is in $Y^+$:
    Add RHS to $Y^+$
Return $Y^+$
Visualizing attribute closure

If LHS is in $Y^+$ and LHS $\rightarrow$ RHS holds, we can add RHS to $Y^+$
$S$ is a set of FDs; $\text{LHS} \rightarrow \text{RHS}$ is a single FD. Return true iff $\text{LHS} \rightarrow \text{RHS}$ follows from $S$.

\begin{align*}
\text{FD\_follows}(S, \text{LHS} \rightarrow \text{RHS}) : \\
Y^+ & = \text{Attribute\_closure}(\text{LHS}, S) \\
\text{return} & \ (\text{RHS} \text{ is in } Y^+) 
\end{align*}
Projecting FDs

◆ Later, we will learn how to normalize a schema by decomposing relations. This is the whole point of this theory.

◆ We will need to know what FDs hold in the new, smaller, relations. We must project our FDs onto the attributes of our new relations.
Example

R(A₁, ..., Aₙ) Set of attributes: A
Decompose into:
- R₁(B₁, ..., Bₖ) Set of attributes: B, and
- R₂(C₁, ..., Cₘ) Set of attributes: C

B \cup C = A, \quad R₁ \bowtie R₂ = R

R₁ = \pi_B(R)
R₂ = \pi_C(R)
$S$ is a set of FDs; $L$ is a set of attributes.

Return the projection of $S$ onto $L$:

all FDs that follow from $S$ and involve only attributes from $L$.

Project($S$, $L$):

Initialize $T$ to $\{\}$. 

For each subset $X$ of $L$:

Compute $X^+$  

Close $X$ and see what we get.

For every attribute $A$ in $X^+$:

If $A$ is in $L$:  

$X \rightarrow A$ is only relevant if $A$ is in $L$ (we know $X$ is).

add $X \rightarrow A$ to $T$.

Return $T$. 
A few speed-ups

- No need to add $X \rightarrow A$ if $A$ is in $X$ itself. It’s a trivial FD.
- These subsets of $X$ won’t yield anything, so no need to compute their closures:
  - the empty set
  - the set of all attributes
- Neither are big savings, but ...
A big speed-up

◆ If we find $X^+ = \text{all attributes}$, we can ignore any superset of $X$.
  ▶ It can only give use “weaker” FDs (with more on the LHS).
◆ This is a big time saver!
Projection is expensive

- Even with these speed-ups, projection is still expensive.
- Suppose $R_1$ has $n$ attributes. How many subsets of $R_1$ are there?
Minimal Basis

◆ We saw earlier that we can very often rewrite sets of FDs in equivalent ways.

◆ Example: $S_1 = \{A \rightarrow BC\}$ is equivalent to $S_2 = \{A \rightarrow B, \ A \rightarrow C\}$.

◆ Given a set of FDs $S$, we may want to find a minimal basis: A set of FDs that is equivalent, but has
  ◆ no redundant FDs, and
  ◆ no FDs with unnecessary attributes on the LHS.
$S$ is a set of FDs. Return a minimal basis for $S$.

**Minimal\_basis(S):**

1. Split the RHS of each FD
2. For each FD $X \rightarrow Y$ where $|X| \geq 2$:
   - If you can remove an attribute from $X$ and get an FD that follows from $S$:
     - Do so! (It’s a stronger FD.)
3. For each FD $f$:
   - If $S - \{f\}$ implies $f$:
     - Remove $f$ from $S$. 
Some comments on minimal basis

◆ Often there are multiple possible results. Depends on the order in which you consider the possible simplifications.

◆ After you identify a redundant FD, you must not use it when computing subsequent closures.
... and less intuitive

◆ When you are computing closures to decide whether the LHS of an FD
  \[ X \rightarrow Y \]
  can be simplified, continue to use that FD.

◆ You must do (2) and (3) in that order.
  Otherwise, must repeat until no changes occur.
Part II:
Using FD Theory to do Database Design
Recall that poorly designed table?

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◆ We can now express the relationships as FDs:
  ◆ part → manufacturer
  ◆ manufacturer → address
  ◆ seller → address
◆ The FDs tell us there can be redundancy, thus the design is bad.
◆ That’s why we care about FDs.
Decomposition

To improve a badly-designed schema $R(A_1, \ldots, A_n)$, we will decompose it into smaller relations $R_1(B_1, \ldots, B_j)$ and $R_2(C_1, \ldots, C_k)$ such that:

1. $R_1 = \pi_{B_1, \ldots, B_j}(R)$
2. $R_2 = \pi_{C_1, \ldots, C_k}(R)$
3. ${B_1, \ldots, B_j} \cup {C_1, \ldots, C_k} = {A_1, \ldots, A_n}$
4. $R_1 \bowtie R_2 = R$
R(A₁, ... Aₙ)  
Set of attributes: A

Decompose into:
- R₁(B₁, ... B_j)  
  Set of attributes: B, and
- R₂(C₁, ... C_k)  
  Set of attributes: C

B ∪ C = A,  
R₁ ⨝ R₂ = R

R₁ = π_B(R)  
R₂ = π_C(R)
But which decomposition?

- Decomposition can definitely improve a schema.
- But which decomposition? There are many possibilities.
- And how can we be sure a new schema doesn’t exhibit other anomalies?
- Boyce-Codd Normal Form guarantees it.
Boyce-Codd Normal Form

◆ We say a relation \( R \) is in **BCNF** if for every nontrivial FD \( X \rightarrow Y \) that holds in \( R \), \( X \) is a superkey.

▷ Remember: *nontrivial* means \( Y \) is not contained in \( X \).

▷ Remember: a *superkey* doesn’t have to be minimal.

◆ [Exercise]
Intuition

In other words, BCNF requires that:

Only things that functionally determine *everything* can functionally determine *anything*.

Why is the BCNF property valuable?

Note:

- FDs are not the problem. They are facts!
- The schema (in the context of the FDs) is the problem.
**R is a relation; F is a set of FDs.**
Return the BCNF decomposition of R, given these FDs.

**BCNF_decomp(R, F):**

If an FD \( X \rightarrow Y \) in \( F \) violates BCNF

Compute \( X^+ \).
Replace \( R \) by two relations with schemas:

\[
R_1 = X^+ \\
R_2 = R - (X^+ - X)
\]

Project the FD’s \( F \) onto \( R_1 \) and \( R_2 \).
Recursively decompose \( R_1 \) and \( R_2 \) into BCNF.

[Example]
1) Start with the LHS of the violating FD.

2) Close the LHS to get one new relation.

3) Everything except the new stuff is the other new relation. $X$ is in both new relations to make a connection between them.
Some comments on BCNF decomp

◆ If more than one FD violates BCNF, you may decompose based on any one of them.
  ▶ So there may be multiple results possible.
◆ The new relations we create may not be in BCNF. We must recurse.
  ▶ We only keep the relations at the “leaves”.
◆ How does the decomposition step help? [Exercise]
Speed-ups for BCNF decomposition

◆ Don’t need to know any keys.
  ▶ Only superkeys matter.

◆ And don’t need to know all superkeys.
  ▶ Only need to check whether the LHS of each FD is a superkey.
  ▶ Use the closure test (simple and fast!).
BCNF

◆ Every attribute depends on:
  ▪ The key
  ▪ The whole key
  ▪ And nothing but the key...

so help me Codd....
More speed-ups

◆ When projecting FDs onto a new relation, check each new FD:
  ◦ Does the new relation violate BCNF because of this FD?
◆ If so, abort the projection.
  ◦ You are about to discard this relation anyway (and decompose further).