These exercises are meant to give you practice devising DFAs and manipulating formal languages.

1. Devise a DFA over the alphabet $\Sigma = \{1, 2, 3\}$ that accepts the language of finite strings that include 321 as a substring.

   (a) Draw the automaton

   ![DFA Diagram](image)

   Sample solution:

   (b) Write down all the parts that define the automaton you’ve drawn (Alphabet $\Sigma$, State space $Q$, transition function $\delta$, etc)

   Sample solution:
   \[
   \Sigma = \{1, 2, 3\} \\
   Q = \{q_0, q_1, q_2, q_3\} \\
   q_0 \text{ is start state} \\
   F = \{q_3\} \\
   \delta:
   \]

   \[
   \begin{array}{cccc}
   1 & 2 & 3 & \\
   q_0 & q_0 & q_3 & q_3 \\
   q_0 & q_2 & q_0 & q_3 \\
   q_1 & q_1 & q_1 & q_3 \\
   \end{array}
   \]

2. Let $\Sigma = \{a, b\}$. Consider the language that consists of all strings that contain neither consecutive a’s nor consecutive b’s. Draw DFA that accepts this language.

   Sample solution:
3. Suppose $L$ is the language of finite binary strings consisting of one or more 1 concatenated with one or more 0. Describe with an English sentence each of the following: Rev($L$), $L^*$, and Rev($L$)$\circ L^*$.

Sample solution:

(a) Rev($L$) is a language that consists of all strings that start with one or more 0s and finish with one or more 1s. For example, 01, 0011, 00001, etc.

(b) $L^*$ is a language that consists of an empty string and strings containing any number of repetitions of the strings that start with one or more 1 concatenated with one or more 0. For example, 1010, 1111000, 101010000, 1110011000010 etc.

(c) Rev($L$)$\circ L^*$ is a language that consists of all strings that start with one or more 0s, followed by one or more 1s, concatenated with any number of repetitions of one or more 1s followed by one or more 0s. For example, 001, 01100, 0110, 001100110010, etc.