Let the alphabet be $\Sigma = \{a, b\}$

1. Are regular expressions $(a + b)^*$ and $a^* + b^*$ equivalent? Explain.
   Solution: Let $R_1 = (a + b)^*$ and $R_2 = a^* + b^*$. $R_1 \neq R_2$, because $R_1$ includes all strings in the alphabet $\Sigma$ while $R_2$ includes repetitions of $a$ (including 0 repetitions) or repetitions of $b$, but no strings that include both $a$ and $b$. The corresponding NFA are different as well:

   ![NFA Diagram](image1)

2. Draw a DFA corresponding to the regular expression $(a + b)(a + b)^*(a^* + b^*)$. Write down the corresponding state invariant that you could use to prove the equality of your DFA to the regular language represented by the provided regexp. You don’t need to provide the proof.
   Solution: First, note that $L((a^* + b^*) \subseteq L(a + b)^*$ and not only that, but any string that can be generated by $(a + b)^*(a^* + b^*)$ can be generated by $(a + b)^*$ due to the definition of the Kleene’s star. Hence, we can simplify $(a + b)(a + b)^*(a^* + b^*) = (a + b)(a + b)^*$. The corresponding DFA is

   ![DFA Diagram](image2)

   State invariants are then as follows:

   $$\delta'(q_0, s) = \begin{cases} q_1 & \text{if } s \text{ starts with an } a \text{ or } b \\ q_0 & \text{otherwise} \end{cases}$$
3. Consider a regexp $R_1$: $a(ba^*)(a^* + b^*)$

(a) Draw an NFA $M_2$ corresponding to the $R_1$ above

Solution: There are many answers to this question, including

![Diagram of NFA M2](image)

The second solution follows from $a(ba^*)(a^* + b^*) = ab(a^*a^* + a^*b^*) = aba^*b^*$

(b) Write down the language $L$ that it represents (a sentence describing all strings)

Answer: $L$ contains all strings that start with $ab$, concatenated with repetitions of $a$ (including zero repetitions of $a$) followed by repetitions of $b$ (including zero repetitions of $b$).

(c) Draw a corresponding DFA

![Diagram of DFA](image)

4. Consider the NFA $A$ with transition relation $\delta = \{(q_0, a, q_1), (q_1, b, q_0), (q_1, b, q_2), (q_2, a, q_0)\}$, with initial state $q_0$ and final states $F = \{q_0\}$. Use the subset construction to find an equivalent DFA.

Answer: Since the initial state of the NFA $A$ is $q_0$ the initial state of the equivalent DFA that we get from the subset construction, call it $B$, is $\{q_0\}$. The state $q_0$ has only one transition $(q_0, a, q_1)$, then we have the transition for $B; (\{q_0\}, a, \{q_1\})$. The state $q_1$ has two transitions $(q_1, b, q_0), (q_1, b, q_2)$, the transition from the state $\{q_1\}$ will be $(\{q_1\}, b, \{q_0, q_2\})$. Similarly, we have the transition relation for the DFA $B$:

$\{(\{q_1\}, a, \{q_1\})\}$

The set of accepting states of the DFA $B$ is:

$F = \{\{q_0\}, \{q_0, q_2\}\}$