Outline

FSAs, continued

Regular Expressions

Regular Languages

Notes
Continue with examples of NFAs

Draw the NFA that recognizes the language $L((a + b) \ast a(a + b)(a + b))$
Do DFA and NFA recognize the same languages? (i.e. are equivalent)
ε-NFAs
Do NFA and $\epsilon$-NFA recognize the same languages? (i.e. are equivalent)
Reminder on language operations

concatenation: \( LL' \) or \( L \cdot L' = \{rt|r \in L, t \in L'\} \).

Special cases:
\( L\{\varepsilon\} = L = \{\varepsilon\}L, \)
\( L\{\} = \{\} = \{\}L. \)

exponentiation: \( L^k \) is concatenation of \( L \) \( k \) times. Special case,
\( L^0 = \{\varepsilon\} \), including \( L^0 = \{\} \quad (!) \)

Kleene star: \( L^* = L^0 \cup L^1 \cup L^2 \cup \ldots. \)
Another way to define languages
In addition to the language accepted by DFSA $L(M)$ and set description $L = \{ \ldots \}$.

Definition: The **regular expressions** (regexps or REs) over alphabet $\Sigma$ is the **smallest** set such that:

1. $\{\}$, $\varepsilon$ are REs over $\Sigma$ (for every $\Sigma$)
2. for every $a \in \Sigma$, $a$ is RE over $\Sigma$
3. if $R$ and $S$ are REs over $\Sigma$, then so are:
   - $R^*$ (star) — highest precedence
   - $RS$ (concatenation) — middle precedence operator
   - $R + S$ (union) — lowest precedence operator
Regular Expression to Language

- $L(\emptyset) = \emptyset$ (the empty language — no strings!)
- $L(\varepsilon) = \{\varepsilon\}$ (the language consisting of just the empty string)
- $L(x) = \{x\}$ (the language consisting of the one-symbol string)

- $L(S + T) = L(S) \cup L(T)$
- $L(ST) = L(S)L(T)$
- $L(T^*) = L(T)^*$
RE Examples

- $L(a + b) = \{a, b\}$
- $L(ab) = \{ab\}$
- $L((a + b)a) = \{aa, ba\} = L(aa + ba)$
- $L(a^*) = \{\epsilon, a, aa, aaaa, \ldots\}$
- $L(aa^*) = \ldots$
- $L((ab)^*) = \ldots$
- $L(a^*b^*) = \ldots$
RE Examples

- $L((a + b)^*) = $

- $L(a^* + b^*) = $

- $L((a + b)(a + b)^*) = $

- All strings of $a$’s and $b$’s that have the same first and last symbol?

- All strings of $a$’s and $b$’s that contain at least 1 $a$?
Do $\varepsilon$-NFA and regular expressions recognize the same languages? (i.e. are equivalent)
Regular Languages