These exercises are intended to give you more practice with recurrence relations.

1. Consider the recurrence relation

\[ T(n) = \begin{cases} 
1 & n = 1 \\
\frac{1}{1 + T\left(\left\lceil \frac{n}{2} \right\rceil\right)} & n > 1 
\end{cases} \]

Prove that \( T(n) \) is non-decreasing.

2. Use repeated substitution (unwinding) to find a closed form for the recurrence \( S \) when \( n \) is a power of 3.

\[ S(n) = \begin{cases} 
1 & n < 3 \\
\frac{1}{n^2 + a_1 S\left(\left\lceil \frac{n}{3} \right\rceil\right) + a_2 S\left(\left\lfloor \frac{n}{3} \right\rfloor\right)} & n > 2 
\end{cases} \]

where \( a_1, a_2 \geq 0 \in \mathbb{N} \) and \( a_1 + a_2 = 3 \).