In the office hour this week, in addition to quick clarifications on questions of Assignments 1, we discussed detailed proof of Example 54 as well as similarities/differences between simple and strong inductions.

- Example 54. We mostly discussed the strong induction proof, with some insights to its simple induction variant. Here, I present a proof by simple induction:

Let $P(n)$ denote $f(n) < 2^{n+2}$ where $f(n) = \begin{cases} 2 & n = 0 \\ 7 & n = 1 \\ 2f(n - 2) + f(n - 1) & n > 1 \end{cases}$

Proof by strong induction.

Basis step. $P(0)$ and $P(1)$ hold as $2 < 2^{0+2}=4$ and $7 < 2^{1+2}=8$, respectively.

Inductive step. Assume $P(i)$ holds for $2 \leq i \leq k$ and arbitrary fixed $k \geq 3 \in \mathbb{N}$; i.e., $f(i) < 2^{i+2}$.

We must show $P(k + 1)$ holds too, i.e., $f(k + 1) < 2^{k+3}$.

$f(k + 1) = 2f(k - 1) + f(k)$ by definition of $f$

$< 2 \cdot 2^{k+1} + 2^{k+2}$ by IH, since $3 \leq k$, $2 \leq k-1 \leq k$

$< 2^{k+2} + 2^{k+2}$

$< 2^{k+3}$

This completes the inductive step. Hence, $f(n) < 2^{n+2}$ $\forall n \in \mathbb{N}$. \qed