CSC236 Intro. to the Theory of Computation
Lecture 4: Recurrences

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http://www.csc.toronto.edu/~csc236h/fall/index.html
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Recurrences

Review

‐ So far
  • Simple Induction, Strong Induction, WOP, and Structural Induction
  • over 48 examples

‐ This week
  • Recurrence relations, closed forms, and proof of their properties

Example 50: rabbits

• A rabbit couple lives in an island. They are newborn and do not breed until they are 2 months old. Since age 2-month, each couple produces another couple per month. Find a recurrence relation for the number of couples after n months, assuming they never die.

Example 50:

Example 51: “00” free strings

• Find a recurrence relation for the number of binary strings of length n that do not contain substring “00”. (Revisit of Example 27.)

Example 52: finding closed form

• Assume $f_n = c_1 f_{n-1} + c_2 f_{n-2}$
  • Find roots of $r^2 - c_2 r - c_1 = 0$, $r_1$ and $r_2$.
  • Then solve $f_n = \alpha_1 r_1^n + \alpha_2 r_2^n$, using the initial values of $f$. 
Example 52:

Example 53:

\[ f(n) = \begin{cases} 
2 & n = 0 \\
7 & n = 1 \\
2f(n - 2) + f(n - 1) & n > 1 
\end{cases} \]

Find the closed form of \( f \).

Example 54:

Prove \( f(n) < 2^{n+2} \). \( f(n) \) defined in Examples 53.

Example 54:

notes: