Consider the following search engine auction problem. There are three slots $a, b, c$ with (respectively) clickthrough rates 4, 3, 1. There are three advertisers $x, y, z$ with (respectively) values per click 7, 5, 1.

1. Suppose the search engine uses the VCG auction.
   - If all advertisers bid truthfully what will be the allocation, the utility to each advertiser and what prices they will pay?

   **Solution** $x$ gets slot $a$ and pays 7; $y$ gets slot $b$ and pays 2; $z$ gets slot $c$ and pays 0.
   - Would any advertiser have an incentive to not bid truthfully?

   **Solution** Since VCG is truthful, there is no incentive for any advertiser to not bid truthfully.

2. Suppose the search engine uses the GSP auction.
   - If all advertisers bid truthfully what will be the allocation, the utility to each advertiser and what prices they will pay?

   **Solution** If everyone bids truthfully, then $x$ gets slot $a$ and pays 20; $y$ gets slot $b$ and pays 3; $z$ gets slot $c$ and pays 0.
   - If all other advertisers bid truthfully, would any advertiser have an incentive to not bid truthfully?

   **Solution** $x$ has incentive to reduce his bid to be less than that of $y$ so as to obtain slot $b$. Specifically, when $x$ bids truthfully, the utility for $x$ is 28-20 = 8. if $x$ bids less than 5 (say 4) then the utility for $x$ is 21 - 3 = 18.