1. (15 points) For each of the following provide an example for a 2 agent game in matrix form for which:

- There is no pure Nash Equilibrium (PNE). Very briefly (e.g. one sentence) explain answer.

  **SOLUTION:** The matching pennies game (see slide 21 of Lecture 9) has no pure NE.

- There is a unique PNE. Is this PNE Pareto optimal? Is it a social optimum?

  **SOLUTION:** The 2 player, 3 action game on slide 13 of Lecture 9 has the unique pure NE \((A, A)\). This NE is a solution optimum and hence must be Pareto optimal. Another example is the prisoners dilemma game (see slide 16 in Lecture 10) where the unique pure NE is neither socially optimal nor Pareto optimal.

- There are at least two PNE. Which of the PNE are Pareto optimal? Which of the PNE are a social optimum?

  **SOLUTION:** The unbalanced coordination game (slide 17 of Lecture 9) has two pure NE. The pure NE \((Uptown, Uptown)\) is socially optimal and Pareto optimal. The pure NE \((Downdown, Downtown)\) is neither socially optimal nor Pareto optimal.
2. (10 points) For each of the following, answer true or false and provide a very brief (e.g. one sentence) explanation.

- In a two agent game, if one agent has a dominant strategy then there must be a PNE.

  SOLUTION: Yes, there must be a pure NE. If say player 1 has a dominant strategy $A$, then player 2 can move to whatever strategy $B$ will maximize the payoff of state $(A, B)$; this state $(A, B)$ is then a pure NE.

- In a three agent game, if one agent has a dominant strategy then there must be a PNE.

  SOLUTION: No, there need not be a Pure NE. Player 3 might have a dominant strategy $C$ which then results in a submatrix (such as the matching pennies matrix) which has no pure NE.