Voting and preference aggregation

CSC200 Lecture 38
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(adapted from Craig Boutilier slides)
Announcements and today's agenda

Today: Voting and preference aggregation
- Reading for next five classes: Ch.23 (plus some important ideas not discussed in the text)
- This week: 23.1-23.6; next week: 23.7-23.10

Announcements
- Final quiz (quiz 8) scheduled for April 1. Aids allowed for the final exam is the same as for all quizzes and tests; namely one 8.5 by 11 sheet (2 sides) of handwritten notes are the only aids allowed.
- As in all assignments, quizzes and tests, you will receive 20% credit for any question (or question part) where you explicitly state “I do not know how to answer this question”.
- Last assignment is due March 30 and has been posted.
CSC200 So Far: Individual Decision Making

- In CSC200 so far, we have discussed *processes by which individuals make their own decisions* and examine the *consequences of these decisions given some surrounding context*.

- Sometimes processes (decisions at individual level) are:
  - implicit (homophily, triadic closure)
  - explicit (game theory, auctions, information and behavior cascades)

- Sometimes we look at consequences of decisions at the:
  - individual level (e.g., games, auctions, small worlds search …)
  - aggregate/network level (e.g., network level behavior like Braess paradox, equilibria, social welfare, direct benefit population effects)

- But sometimes a *single decision* must be made so as to apply to an *entire group of individuals*.
A Simple Example

- City has budget to build one new recreational facility: three options

- Three legislators differ in preferences over the options

- How do we decide when we have to:
  - choose a single consensus alternative?
  - rank all three alternatives?
Voting and Preference Aggregation

- Some examples of single decisions for a group/population
  - group of friends deciding on a club, restaurant, vacation, …
  - group of businesses (or in the era of Groupon, consumers) choosing a supplier for a specific item to generate a volume discount
  - city deciding on location of new park, new bus routes, etc…
  - hiring committee selecting a job candidate
  - company designing a new product for a target market
  - search engine returning (non-personalized) search results for query $q$
  - recommender system: (non-personalized) ordering of movies, music, …
  - government setting economic, social, environmental policy
  - … of course, electing political representatives to some legislative body

- What’s so difficult about this?
  - People have different preferences (don’t agree on the best choice)
  - Need some notion of \textit{compromise, consensus} or \textit{group-satisfaction} to select an alternative
Social choice: study of collective decision making

Aggregation of individual preferences determines a consensus outcome for some population

- Political representatives, committees, public projects,…
- Studied for millennia, formally for centuries
- Increasing importance for low stakes domains

Key points:
- we aggregate preferences, not judgments/opinions (for now)
  - we’ll see connections to info aggregation (Ch.22)
- preferences are qualitative: rankings, not utilities or valuations
  - looks like mechanism design (e.g., for designing auctions) but without valuations and monetary transfers
  - can be difficult to compare, add, average preferences
Individual Preferences

- Assume a finite set of alternatives $A$ (e.g., rec facilities)
- A person’s preferences is a *total linear ordering* (ranking) of $A$
  - Picture is the same as when we discussed Gale-Shapley matching

- Ordering is equivalent to requiring that a person’s preference be:
  - **complete:** everything comparable; either $a > b$ or $b > a$ for any $a, b$ in $A$
  - **transitive:** if $a > b$ and $b > c$, then $a > c$

- Completeness important (though allowing ties is reasonable)
  - otherwise when faced with two choices $\{a, b\}$, person is unable to decide

- Transitivity important to prevent cyclic (strict) preferences
  - violates certain rationality principles (e.g., the “money pump”)
Voting Systems

Assume:
- $m$ alternatives $A = \{a_1, \ldots, a_m\}$
- $n$ individuals or voters $N = \{1, \ldots, n\}$ with preferences over $A$

A *voting system* or *rule* accepts the preferences of $N$ as input and aggregates them to determine either:
- a *winner* or consensus alternative from $A$
- a *group/consensus ranking (or top k ranking)* of the alternatives
- Note: approval voting doesn’t quite fit this definition

This is a broad definition! How do we go about choosing a reasonable voting rule?
- Let’s focus on picking winners for now (not rankings)
- Let’s start by looking at a few examples
Plurality Voting

- **Plurality voting:**
  - **Input:** rankings of each voter
  - **Winner:** alternative ranked 1\textsuperscript{st} by greatest number of voters
    - number of 1\textsuperscript{st}-place rankings is a’s *plurality score*
    - *complete* rankings not needed, just votes for most preferred alternatives
      - we’ll ignore ties for simplicity
  - This is a most familiar scheme, used widely:
    - locally, provincially, nationally for electing political representatives
  - With only 2 alternatives, often called *majority voting*

- Example preference profile (three alternatives):
  - A > B > C: 5 voters
  - C > B > A: 4 voters
  - B > C > A: 2 voters

- **Winner:** A wins (plurality scores are A: 5; C: 4; B:2)
The Borda Rule

- **Borda voting rule:**
  - **Input:** rankings of each voter
  - **Borda score** for each alternative \( a \): \( a \) gets \( m-1 \) points for every 1\(^{st}\)-place rank, \( m-2 \) points for every 2\(^{nd}\)-place, etc.
  - **Winner:** alternative with highest Borda score
  - Used in sports (Heismann, MLB awards), variety of other places
  - Proposed by Jean-Charles, chevalier de Borda in 1770 to elect members to the French Academy of Sciences (also Ramon Llull, 13\(^{th}\) century)

- **Example profile** (three alternatives, positional scores of 2, 1, 0):
  - \( A > B > C \): 5 voters
  - \( C > B > A \): 4 voters
  - \( B > C > A \): 2 voters

- **Winner:** B wins (Borda scores are: \( B: 13; \ A: 10; \ C: 10 \))
  - Notice: more sensitive to *the entire range of preferences* than plurality (which ranked \( B \) last)
Approval Voting

- Approval Voting
  - Input: voters specify a *subset* of alternatives they “approve of”
  - Approval score: a point given to a for each approval
    - variant: *k*-approval, voter lists exactly *k* candidates
  - Winner: alternative with highest approval score
  - used in many informal settings (at UN, Doge of Venice, …)
  - Steven Brams a major advocate (see Wikipedia article)

- Example profile (three alternatives, approvals in bold):
  - **A > B > C**: 5 voters (approve of only top alternative)
  - **C > B > A**: 4 voters (approve of only top alternative)
  - **B > C > A**: 2 voters (approve of top two alternatives)

**Winner:** C wins (approval scores are: C: 6; A: 5; B: 2)
- Notice: can’t predict vote based on ranking alone!
Positional Scoring (Voting) Rules

- Observe that plurality, Borda, $k$-approval, $k$-veto are all each *position scoring rules*
- Each assigns a score $\alpha(j)$ to each rank position $j$
  - almost always non-increasing in $j$
- The winner is the candidate $a$ with max total score: $\sum_i \alpha(r_i(a))$

In general: $a(1)$ $a(2)$ $a(3)$ $a(4)$

| Plurality: | 1 | 0 | 0 | 0 |
| Borda:     | 3 | 2 | 1 | 0 |
| 2-Approval:| 1 | 1 | 0 | 0 |
| 1-Veto:    | 1 | 1 | 1 | 0 |

Could be: 10 2 0 0
Which of these is Better?

- Notice that on the same vote profile, plurality, Borda, and approval gave different winners!

- Which is best?
  - hard to say: depends on social objective one is trying to meet
  - common approach: identify axioms/desirable properties and try to show certain voting rules satisfy them
    - we will see it is not possible in general to satisfy all axioms!

- But let’s look at a few more voting rules just to get a better sense of things.
There are Hundreds of Voting Rules

**Single-transferable vote (STV) or Hare system**

- Round 1: vote for favorite candidate; eliminate candidate with lowest plurality score;
- Round $t$: if your favorite is eliminated at round $t-1$, recast vote for favorite remaining candidate; eliminate candidate with lowest plurality score;
- Round $m-1$: winner is last remaining candidate if not chosen sooner
  - terminate at any round if plurality score of top candidate is at least $n/2$ (i.e., there is a majority winner)
- Used: Australia, New Zealand, Ireland, some political party conventions
  - Doesn’t necessitate repeated voting: voters can submit rankings once
- When would this be a bad voting rule?

**Nanson’s rule**

- Just like STV, but use Borda score to eliminate candidates
There are Hundreds of Voting Rules

- **Egalitarian (maxmin fairness)**
  - Winner maximizes minimum voter’s rank: \( \text{argmax}_a \min_j (m-r_j(a)) \)

- **Copeland**
  - Let \( W(a,b,r) = 1 \) if more voters rank \( a \succ b \); 0 if more \( b \succ a \); \( \frac{1}{2} \) if tied
  - Score \( s_c(a,r) = \sum_b W(a,b,r) \); winner is \( a \) with max score
    - *i.e., winner is candidate that wins most pairwise elections*

- **Tournament/Cup**
  - Arrange a (usually balanced) tournament tree of pairwise contests
  - Winner is last surviving candidate
  - We’ll discuss this in more detail later
Condorcet Principle

- How would you determine “societal preference” between a pair of alternatives \(a\) and \(b\)?

- A natural approach: run a “pairwise” majority vote: if a *majority* of voters prefer \(a\) to \(b\), then we say *the group prefers \(a\) to \(b\)*

- **Condorcet winner**: an alternative that beats every other in a pairwise majority vote
  - proposed by Marie Jean Antoine Nicolas de Caritat, marquis de Condorcet in 1785
  - if there is a Condorcet winner, it must be unique
  - a rule is *Condorcet-consistent* if it selects the Condorcet winner (if one exists)

- Condorcet winners need not exist (next slide)
  - and many natural voting rules are not Condorcet consistent (e.g., plurality, Borda, STV are not), but some are: Nanson, Copeland, Cup, etc.
Condorcet Paradox

**Condorcet paradox:**

- suppose we use the pairwise majority criterion to produce a societal preference ranking
- pairwise majority preferences may induce cycles in societal ranking (i.e., the preference ranking is not transitive)

**Simple example:**

- \( A > B > C \): \( m/3 \) voters
- \( C > A > B \): \( m/3 \) voters
- \( B > C > A \): \( m/3 \) voters
- Societal ranking has \( A > B, B > C, \) and \( C > A \) (!)
- No clear way to produce a consensus ranking
- Also evident that this preference profile has no Condorcet winner
Violations of Condorcet Principle

- Plurality violates Condorcet
  - 499 votes: $A > B > C$
  - 3 votes: $B > C > A$
  - 498 votes: $C > B > A$
  - plurality chooses $A$; but $B$ is a CW ($B > A$ 501:499; $B > C$ 502:498)

- Borda violates Condorcet
  - 3 votes: $A > B > C$
  - 2 votes: $B > C > A$
  - 1 vote: $B > A > C$
  - 1 vote: $C > A > B$
  - Borda chooses $B$ (9 pts) ; but $A$ is a CW ($A > B$ 4:3; $A > C$ 4:3)
  - notice *any positional* scoring rule (not just Borda) will choose $B$ if scores strictly decrease with rank
The Axiomatic Method

- Considerable work studies various "axioms" or principles that we might like voting rules to satisfy and asks whether we can devise rules that meet these criteria.

- For example, the Condorcet principle is an axiom/property we might consider desirable. We’ve seen some voting rules satisfy it, and others do not.

- Next time we’ll consider a few more rather intuitive axioms.