CSC165 Tutorial #2

Exercises

Winter 2015

Work on these exercises before the tutorial. You don’t have to come up with a complete solution, but you should be prepared to discuss them with your TA.

1. Consider the statements below:

\[ \forall n \in \mathbb{N}, P(n) \Rightarrow Q(n) \]
\[ \exists n \in \mathbb{N}, P(n) \Rightarrow Q(n) \]
\[ \forall n \in \mathbb{N}, Q(n) \Rightarrow P(n) \]
\[ \exists n \in \mathbb{N}, Q(n) \Rightarrow P(n) \]

Evaluate each of the four sentences above using each of the four definitions of predicates \( P \) and \( Q \) below. Briefly explain your evaluation of each of the sixteen cases.

(a) \( P(n) : n < 0 \) \quad \( Q(n) : n^2 < 0 \)
(b) \( P(n) : n < 0 \) \quad \( Q(n) : n^2 > 17 \)
(c) \( P(n) : n > 3 \) \quad \( Q(n) : n^2 < 0 \)
(d) \( P(n) : n > 3 \) \quad \( Q(n) : n^2 > 17 \)

2. Consider the statement:

\( S_1 : \) For all students, missing an assignment or missing a quiz guarantees not getting a 100% in CSC165.

(a) Write \( S_1 \) in logical notation. The logical statement must be well-formed. Define all sets and predicate symbols that you use in the logical statement.
(b) Suppose \( S_1 \) is true and a student didn’t miss any assignments. What, if anything, can be determined about the student missing quizzes and getting 100% in CSC165? Briefly justify your answer.
(c) Suppose \( S_1 \) is true and a student got 100% in CSC165. What, if anything, can be determined about the student missing assignments and quizzes? Briefly justify your answer.

3. Translate the following sentences into logical notation. The logical sentences must be well-formed. Define all sets and predicate symbols that you use in the logical sentences.

(a) If some NP-complete problem can be solved efficiently, then every NP-complete problem can be solved efficiently.
(b) Some courses have exactly one prerequisite course.
(c) Some courses have the same prerequisite courses.
(d) For every person, being a student is necessary for enrollment.

4. Give the contrapositive and converse of Question 3(d) in English.