This test consists of 4 questions on 6 pages (including this one). When you receive the signal to start, please make sure that your copy of the test is complete.

Please answer questions in the space provided.

Good Luck!
Question 1. [14 marks]

- Express each of the following sentences in logical notation.
  Define all set and predicate symbols that you use in the logical expressions.

- Write the negation of each of the sentences in English and in logical form.
  Simplify the logical sentences so that only predicates are negated.

All logical sentences must be well-formed.

Part (a) [3 marks] At least one of your friends is perfect.
Solution:
Let \( F \) be the set of all your friends, and \( \text{Perfect}(x) \) denotes \( x \) is perfect.
\[
\exists x \in F, \text{Perfect}(x)
\]
Negation:
\[
\forall x \in F, \neg \text{Perfect}(x)
\]
All of your friends are not perfect.

Part (b) [5 marks] Students graduate if they pass all courses.
Solution:
Let \( S \) be the set of all students, \( C \) be the set of all courses, \( \text{Graduate}(x) \) denotes \( x \) graduated and \( \text{Pass}(x, y) \) denotes \( x \) has passed \( y \).
\[
\forall x \in S, (\forall y \in C, \text{Pass}(x, y)) \Rightarrow \text{Graduate}(x)
\]
Negation:
\[
\exists x \in S, \forall y \in C, \text{Pass}(x, y) \wedge \neg \text{Graduate}(x)
\]
Some students passed all courses but have not graduated.

Part (c) [3 marks] Everyone in CSC165 has studied a foreign language.
Solution:
Let \( S \) be the set of all students in CSC165, and \( F(x) \) denotes \( x \) has studied a foreign language.
\[
\forall x \in S, F(x)
\]
Negation:
\[
\exists x \in S, \neg F(x)
\]
Some students in CSC165 have not studies a foreign language.

Part (d) [3 marks] Each person in the race participated in at least one competition.
You must only use the following symbols in your logical sentence:
\( P \): the set of all persons in the race.
\( C \): the set of all competitions.
\( \text{Par}(x, y) \): \( x \) participated in competition \( y \).
Solution:
\[
\forall x \in P, \exists y \in C, \text{Par}(x, y)
\]
Negation:
Some persons in the race did not participate in any competitions.

**Question 2.** [6 marks]

Let $V(x, y)$ denotes $x$ has visited $y$, $S$ denotes the set of all students in CSC165, and $R$ denotes the set of all stores.

Express each of the statements by a simple English sentence.

Avoid symbols (e.g. $x$) and predicates (e.g. $T(x,y)$) in English sentences.

**Part (a) [1.5 mark]**

$\exists y \in R, V(\text{Ella}, y)$.

Solution: Ella visited some store.

**Part (b) [1.5 mark]**

$\exists x \in S, \forall y \in R, V(x, y)$

Solution: Some CSC165 student visited all stores.

**Part (c) [3 marks]**

$\exists x \in S, \exists y \in S, \forall z \in R, (x \neq y) \land (V(x, z) \iff V(y, z))$

Solution: At least two different CSC165 students visited exactly the same stores.
Question 3. [10 Marks]

Verify if the following arguments are logically valid. Provide logical justifications for your answers.
(Hint: it might be helpful if you re-state the arguments in symbolic notation)

Part (a) [4 Marks]

Given the following assumption:
AS1: There is no alive dinosaur.

we can conclude that:
Con1: Alive dinosaurs eat humans.

Solution:
D: set of all dinosaurs.
L(x): x is alive.
E(x): x eats humans.

AS1 can be re-stated as ¬∃x ∈ D, L(x).

Con1 can be re-stated as ∀x ∈ D, L(x) ⇒ E(x).

The argument is valid because Con1 is vacuously true.

Part (b) [6 Marks]

Given the following assumptions:
AS2: A decent education is necessary for getting a job.
AS3: You have a decent education.

we can conclude that:
Con2: You will get a job.

Solution:
P: Having a decent education.
Q: Getting a job.

AS2 can be re-stated as Q ⇒ P.
AS3 can be re-stated as P.
Con2 can be re-stated as Q.

The argument is invalid, since given Q ⇒ P and P, it is not logically possible to conclude Q.
Question 4. [20 marks]

For each statement below, identify whether it is satisfiable, unsatisfiable, or is a tautology, and prove your answer.

To prove that a statement is a tautology you must use manipulation rules (justify each step of your derivation by naming the rule). Use truth tables to justify satisfiability or unsatisfiability.

Part (a) [9 marks]

\((P \Rightarrow Q) \land (P \Rightarrow R) \Rightarrow (Q \Leftrightarrow R)\)

Solution: The statement is satisfiable.

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Part (b) [11 marks]

\((P \Rightarrow Q) \land ((P \land Q) \land R) \Rightarrow R\)

Solution: The statement is a tautology.

\[
\begin{align*}
((P \Rightarrow Q) \land ((P \land Q) \land R) \Rightarrow R) & \Leftrightarrow \neg((\neg P \lor Q) \land ((P \land Q) \land R)) \lor R \quad \text{(Implication Rule)} \\
& \Leftrightarrow ((P \land \neg Q) \lor ((\neg P \lor \neg Q) \lor \neg R)) \lor R \quad \text{(DeMorgan's law)} \\
& \Leftrightarrow (((P \land \neg Q) \lor (\neg P \lor \neg Q)) \lor (\neg R \lor R)) \quad \text{(Associativity)} \\
& \Leftrightarrow (((P \land \neg Q) \lor (\neg P \lor \neg Q)) \lor True) \quad \text{(Def. of \lor)} \\
& \Leftrightarrow True \quad \text{(Def. of \lor)}
\end{align*}
\]
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# 1: _____/14
# 2: _____/ 6
# 3: _____/10
# 4: _____/20

TOTAL: _____/50