Announcements:
  • PS 0, PS 1
  • be sure to read CourseNotes (start Ch 2 Fri)
  • may review other instructor class notes too

Last Time:
  \( \forall, \exists \) predicates

Consider predicate:
  Loves \((x, y)\) : "person \(x\) loves person \(y\)",
  where \(x, y\) are in \(\mathcal{H}\)
\(\mathcal{H}: \) the set of humans

How could we express:
  • "Everyone loves everyone."
\[ \forall x, y \in \mathcal{H}, \text{Loves}(x, y) \]
  • "Someone loves someone."
\[ \exists x, y \in \mathcal{H}, \text{Loves}(x, y) \]
  • "Everyone loves someone."
\[ \forall x \in \mathcal{H}, \exists y \in \mathcal{H}, \text{Loves}(x, y) \]
possible for two people to love same person:

\[ \forall x \exists y \forall z. L(x, y) \land L(x, z) \]

\[ L(x, y) \land L(x, \text{Diane}) \land L(x, \text{Tom}) \land \text{Loves(Catherine)} \]

Is this equivalent to:

\[ \exists y \in H, \forall x \in H, \text{Loves}(x, y) \]

This says: "There is someone who is loved by everyone."

Can conclude: for an arbitrary predicate \( P \):

\[ (\forall x \in H, \exists y \in H, P(x, y)) \Rightarrow (\exists y \in H, \forall x \in H, P(x, y)) \]

is false

Consider the converse:

\[ (\exists y \in H, \forall x \in H, \text{Loves}(x, y)) \Rightarrow (\forall x \in H, \exists y \in H, \text{Loves}(x, y)) \]

There is someone who is loved by everyone. Does it follow that: "Everyone loves someone."

In general:
\[(\exists y \in D, \forall x \in D, P(x,y)) \Rightarrow (\forall x \in D, \exists y \in D, P(x,y))\]

**Negation**:

\[\forall x \in D, P(x)\]

says "every \(x \in D\) is such that \(P(x)\) is True."

\[\neg (\forall x \in D, P(x))\]

says "not every \(x \in D\) is such that \(P(x)\) is True."

i.e., some \(x \in D\) is such that \(P(x)\) is False.

\[\neg P(x)\] is True

\[\neg (\forall x \in D, P(x)) \iff (\exists x \in D, \neg P(x))\]

\[\exists x \in D, P(x)\]

says "some \(x \in D\) is such that \(P(x)\) is True."

\[\neg (\exists x \in D, P(x))\]

says "no \(x \in D\) is such that \(P(x)\) is True."

i.e., every \(x \in D\) is such that \(\neg P(x)\) is True.

\[\neg (\exists x \in D, P(x)) \iff (\forall x \in D, \neg P(x))\]
Employee database.

domain E: set of employees.
predicates: O(x): "employee x earns over $50,000", where x ∈ E
F(x): "employee x is female", where x ∈ E

How to express:
"All employees earning over $50,000 are female".

using predicate logic:

∀ e ∈ E, O(e) ⇒ F(e)

for all employees

[ ∀ e ∈ E, O(e) ∧ F(e)

says "all employees earn over $50,000 and are female"

different from "(*)"

- does F(e) result from O(e) or must it be true?

Negating:
Not all employees earning over $50,000 are female.
\[ \neg (\forall e \in E, \ 0(e) \Rightarrow F(e)) \]

or

\[ \exists e \in E, \ \neg (0(e) \Rightarrow F(e)) \]

How to negate an implication?

De Morgan:

\[ \neg (p \lor q) \iff \neg p \land \neg q \]
\[ \neg (p \land q) \iff \neg p \lor \neg q \]

\[ (p \Rightarrow q) \iff (\neg p \lor q) \quad \text{(Implication equivalence)} \]

\[ \neg (p \Rightarrow q) \iff \neg (\neg p \lor q) \quad \text{(Implication)} \]
\[ \iff \neg \neg p \land \neg q \quad \text{(DeMorgan)} \]
\[ \iff p \land \neg q \quad \text{(Double negation)} \]

back to problem

\[ \neg (\forall e \in E, \ 0(e) \Rightarrow F(e)) \]

\[ \iff \exists e \in E, \ \neg (0(e) \Rightarrow F(e)) \]

\[ \iff \exists e \in E, \ 0(e) \land \neg F(e) \]

"Some non-female employee makes over 50,000."
Define divisibility.

Let \( n, d \in \mathbb{Z} \). We say that \( d \) divides \( n \), or \( n \) is divisible by \( d \), if and only if there exists a \( k \in \mathbb{Z} \) such that \( n = dk \).

notation: \( d \mid n \) to represent \( d \) divides \( n \)

\( \mid \) is a binary predicate similar to \( = \) or \( < \)

\( 3 \mid 6 \) is True

\( 3 \nmid 7 \) is False

"If an integer divides 10 then it also divides 100."

\[ \forall x \in \mathbb{Z}, \ x \mid 10 \Rightarrow x \mid 100 \]

\[ \Rightarrow \ \forall x \in \mathbb{Z}, \ (\exists k \in \mathbb{Z}, \ 10 = x \cdot k) \Rightarrow (\exists m \in \mathbb{Z}, \ 100 = x \cdot m) \]