Chapter 6: Graphs and Trees

\[ \text{Defn} \]
A graph is a pair of sets
\[ G = (V, E) \], where

- \( V \) is a set, called the vertices of the graph
- \( E \) is a set of pairs of vertices.
$E = \{ \{u_1, v_1\}, \{u_2, v_2\}, \{u_3, v_3\}, \ldots \}$, where each $u_i, v_i \in V$.

Each pair is an edge of the graph.

- $E$ could be empty.
- Edges have no direction. $\{u, v\} = \{v, u\}$
- No "loops": $\{u, u\}$, $u \neq v$
- Edges are all the same

\underline{Theorem} For all graphs $G = (V, E)$,

$|E| \leq \frac{|V|(|V|-1)}{2}$.

\underline{Proof idea}
Proof idea

The max # of edges =
the # of size two subsets of V.

Question: Can you always get from one vertex to another?

Defn Let $G = (V, E)$.

Let $u, v \in V$.

We say $u$ and $v$ are adjacent, or are neighbours, when $\{u, v\} \in E$.

A path between $u$ and $v$ is a sequence of vertices $v_0, v_1, v_2, \ldots, v_k$
where:

1. $v_0 = u$ and $v_k = v$

2. For each $i$, $v_i$ and $v_{i+1}$ are adjacent

3. There are no duplicates in the sequence.

The length of the path is the number of edges ("k" in the above notation).

The distance between 2 vertices is the length of the shortest path between them.

We say $u$ and $v$ are connected when there exists a path between them.

We say $G$ is connected when
We say $G$ is connected when 
\[ \forall u, v \in V, u \text{ and } v \text{ are connected} \]