Outline

- general trees continued...
- binary trees
- traversals
- binary search trees
traversal

The functions and methods we have seen get information from every node of the tree — in some sense they traverse the tree.

Sometimes the order of processing tree nodes is important: do we process the root of the tree (and the root of each subtree...) before or after its children? Or, perhaps, we process along levels that are the same distance from the root?
def preorder_visit(t, act):
    """
    Visit each node of Tree t in preorder, and act on the nodes as they are visited.
    
    @param Tree t: tree to visit in preorder
    @param (Tree)->Any act: function to carry out on visited Tree node
    @rtype: None
    
    >>> t = descendants_from_list(Tree(0),
                      [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> preorder_visit(t, act)
    0
    1
    4
    5
    6
    2
    7
    3
    """
def postorder_visit(t, act):
    """
    Visit each node of t in postorder, and act on it when it is visited.
    
    @param Tree t: tree to be visited in postorder
    @param (Tree)\rightarrow Any act: function to do to each node
    @rtype: None
    """

    >>> t = descendants_from_list(Tree(0), [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> postorder_visit(t, act)
    4
    5
    6
    1
    7
    2
    3
    0
    """
def levelorder_visit(t, act):
    """
    Visit every node in Tree t in level order and act on the node as you visit it.
    
    @param Tree t: tree to visit in level order
    @param (Tree)->Any act: function to execute during visit
    
    >>> t = descendants_from_list(Tree(0),
                               [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> levelorder_visit(t, act)
    0
    1
    2
    3
    4
    5
    6
    7
    """
You may have noticed in the last slide there were no recursive calls, and a queue was used to process a recursive structure in level order.

Careful use of a stack allows you to process a tree in preorder.
traversal tracing...
refactor

- make flatten and height methods

- tweak __str__ and __repr__ so that they’ll work better for binary trees

- subclass Tree to BinaryTree
tree inheritance issues

one approach to BinaryTree would be to make it a subclass of Tree, but there are some design considerations

▶ any client code that uses Tree would be required not to violate the branching factor (2) of BinaryTree

▶ one way to achieve this would be to make Tree immutable: make sure there is no way to change children or value, and then have subclasses that might be mutable

we will take a different approach: a completely separate BinaryTree class
Change our generic Tree design so that we have two named children, left and right, and can represent an empty tree with None

class BinaryTree:
    ""
    A Binary Tree, i.e. arity 2.
    ""

    def __init__(self, value, left=None, right=None):
        ""
        Create BinaryTree self with value and children left and right.
        """
        @param BinaryTree self: this binary tree
        @param object value: value of this node
        @param BinaryTree|None left: left child
        @param BinaryTree|None right: right child
        @rtype: None
        """
        self.value, self.left, self.right = value, left, right
special methods...

We’ll want the standard special methods:

- `__eq__`
- `__str__`
- `__repr__`
you've implemented contains on linked lists, nested Python lists, general Trees before; implement this function, then modify it to become a method

```python
def contains(node, value):
    """
    Return whether tree rooted at node contains value.

    @param BinaryTree|None node: binary tree to search for value
    @param object value: value to search for
    @rtype: bool
    
    >>> contains(None, 5)
    False
    >>> contains(BinaryTree(5, BinaryTree(7), BinaryTree(9)), 7)
    True
    """
```
Binary arithmetic expressions can be represented as binary trees:

```
3.0

4.0

'+'

'*

7.0

3.0

4.0
```
evaluating a binary expression tree

- there are no empty expressions
- if it’s a leaf, just return the value
- otherwise...
  - evaluate the left tree
  - evaluate the right tree
  - combine left and right with the binary operator

Python built-in `eval` might be handy.
A recursive definition:

- visit the left subtree \texttt{inorder}
- visit this node itself
- visit the right subtree \texttt{inorder}

The code is almost identical to the definition.
- visit this node itself
- visit the left subtree in **preorder**
- visit the right subtree in **preorder**
postorder

- visit the left subtree in **postorder**
- visit the right subtree in **postorder**
- visit this node itself
level order

- visit root
- visit root’s children
- visit root’s grandchildren
- visit root’s greatgrandchildren
- ...

definition

Add ordering conditions to a binary tree:

- data are comparable
- data in left subtree are less than node.data
- data in right subtree are more than node.data
why binary search trees?

Searches that are directed along a single path are efficient:

- a BST with 1 node has height 1
- a BST with 3 nodes may have height 2
- a BST with 7 nodes may have height 3
- a BST with 15 nodes may have height 4
- a BST with $n$ nodes may have height $\lceil \lg n \rceil$. 
bst_contains

If node is the root of a “balanced” BST, then we can check whether an element is present in about \( \log n \) node accesses.

```python
def bst_contains(node, value):
    """
    Return whether tree rooted at node contains value.

    Assume node is the root of a Binary Search Tree

    @param BinaryTree|None node: node of a Binary Search Tree
    @param object value: value to search for
    @rtype: bool
    
    >>> bst_contains(None, 5)
    False
    >>> bst_contains(BinaryTree(7, BinaryTree(5), BinaryTree(9)), 5)
    True
    """

    # use BST property to avoid unnecessary searching
```
def insert(node, data):
    """
    Insert data in BST rooted at node if necessary, and return new root.

    Assume node is the root of a Binary Search Tree.
    """
    @param BinaryTree|None node: root of a binary search tree.
    @param object data: data to insert into BST, if necessary.

    >>> b = BinaryTree(8)
    >>> b = insert(b, 4)
    >>> b = insert(b, 2)
    >>> b = insert(b, 6)
    >>> b = insert(b, 12)
    >>> b = insert(b, 14)
    >>> b = insert(b, 10)
    >>> print(b)
    14
    12
    10
    8