overview

In assignment 2, you will build on the foundation laid out in assignment 1 in order to add a new game, called Stonehenge, and a new strategy, called minimax (for which you are to implement an iterative version and a recursive version). Starter code has been provided so that you should be able to “plug in” your new classes for games, and new strategies, without re-writing existing code. By the end of this assignment, you’ll have a new game and a much stronger opponent to play against.

Stonehenge

Stonehenge is played on a hexagonal grid formed by removing the corners from a triangular grid. Boards can have various sizes based on their side-length (the number of cells in the grid along the bottom), but are always formed in a similar manner: For side-length $n$, the first row has 2 cells, and each row after has 1 additional cell up until there’s a row with $n + 1$ cells, after which the last row has only $n$ cells in it.

Players take turns claiming cells (in the diagram: circles labelled with a capital letter). When a player captures at least half of the cells in a ley-line (in the diagram: hexagons with a line connecting it to cells), then the player captures that ley-line. The first player to capture at least half of the ley-lines is the winner. A ley-line, once claimed, cannot be taken by the other player.

For your implementation of Stonehenge, you must fulfill the following specifications in order for us to test your code:
• `str_to_move()` should take in a capital letter ("A", "B", etc.).

• Cells should either be labelled with a capital letter ("A", "B", etc.) if they're unclaimed, or 1 or 2 if a player has claimed that cell.

• Ley-lines should be marked with an @ if they're unclaimed, or 1 or 2 if it has been claimed.

• Ley-line markers for rows must precede the cells in that row (e.g. @ - A - B as opposed to A - B - @)

• Ley-line markers for down-left diagonals (/) are placed either:
  - Before any rows of cells for 2 of the down-left diagonals (the ones connected to A and B)
  - Or in the row of cells above the row they're connected to, following all of the cells in that row.

• Ley-line marks for down-right diagonals (\) are placed either:
  - In the very last row (after every row containing cells) in order of the cells they’re connected to
  - Or in the last row of cells following the very last cell, below the row containing the cell it’s connected to.

**minimax**

Minimax is a strategy that picks the move that minimizes the possible loss for a player, working along the lines of “if my opponent were to play perfectly, which move of mine would provide the lowest score for them?.” In the case of our games, where the only scores are ‘winning’ (i.e. a score of 1) or ‘losing’ (a score of -1), this becomes “is there a move that can guarantee a win no matter what the opponent does?”

For this assignment, you are to implement 2 versions of minimax: one implemented recursively, and the other implemented iteratively.

**recursive minimax**

For recursive minimax, we try to find a move that produces a “highest guaranteed score” at each step for the current player. For a game state that’s over, the score is:

• 1 if the current player is the winner
• -1 if the current player is the loser
• 0 if the game is a tie

If the state isn’t over, then we take a move that guarantees the “highest guaranteed score” accessible from the available moves. To do this, we get the scores that our opponent can get from each move, and get their opposite (multiply them by -1): after all, if your opponent can guarantee a score of 1 for themself (i.e. a win), then that means you would be guaranteed a score of -1 (i.e. a loss).

For example, for some random game, suppose you have 3 possible moves: One that results in state A, the other in state B, and the last in state C.

Now suppose your opponent can guarantee a score of -1 for themselves from state A, a score of 1 from state B, and a score of 0 from state C. Then, equivalently, that means you can guarantee a score of 1 for yourself if you make the move to state A, -1 from state B, and 0 from state C. Thus, the “highest guaranteed score” for your current state would be 1, so you want the move that takes you to state A.
iterative minimax

For iterative minimax, the logic is similar to recursive minimax. However, you can't use recursion to go through various states: you have to use a loop of some sort, and also keep track of your states and the best values reachable from each of them. To do this, you will likely need to use a stack (to keep track of which states you’ve yet to explore) and a tree structure (to keep track of your various game states).

We start with our current game state and wrap it in a tree-node-like structure: this should keep track of the children (i.e. game states reachable from this one), and the “highest guaranteed score” reachable from this state. Initially, both the “highest guaranteed score” and the children should be unknown/not yet set. Then, we add it to our stack.

We progressively remove the tree-like structures from the stack. If the state is over, then the “highest guaranteed score” is:

- 1 if our player is the winner
- -1 if our player is the loser
- 0 if the game is a tie

If the state is not over, then we try the following:

- If there are no known children, then we make a tree-node-like structure for each of the states accessible from our available moves and set those as our children. We then add each of these to our stack after the parent node.

- If there are children already, then we've already seen this structure recently. Since we're in a stack, that means all of the children have been examined too, so they should all have found their “highest guaranteed score”. So, we look at the scores of each of the children, and set the “highest guaranteed score” to the maximum of the opposite of those (as we did in recursive minimax).

Once the stack is empty, we've examined all possible states, and our original state should know its “highest guaranteed score” as well.

your job

- We have provided you with:
  - game.py, a superclass for games.
  - game_state.py, a superclass for game states.
  - game_interface.py, code to use your game classes.
  - strategy.py, a module for strategies.
  - subtract_square_game.py, a text-based subtract square game class, subclass of Game.
  - subtract_square_state.py, a text-based subtract square game state, subclass of GameState.
  - a2.pyta.txt, a configuration file for python_ta
  - stonehenge_unittest_basic.py, some unit tests to increase your confidence that you are on the right track with implementing the Stonehenge game.
  - minimax_unittest_basic.py, some unit tests to increase your confidence that you are on the right track implementing minimax.
Copy these into a subdirectory of your project, where you will work on your code.

- Implement classes `StonehengeGame` (subclass of `Game`) and `StonehengeState` (subclass of `GameState`) to implement the game Stonehenge, and save them in `stonehenge.py`.

- Implement strategies `recursive_minimax_strategy(game)` and `iterative_minimax_strategy(game)` each of which provide a move that guarantees the highest possible score from the current position.

- Be sure to have:

  ```python
  if __name__ == "__main__":
      from python_ta import check_all
      check_all(config="a2_pyta.txt")
  ```

  ... at the bottom of each of the files you submit.

Submitting your work

Submit all your code on MarkUs by 10 p.m. March 6th. Click on the “Submissions” tab near the top. Click “Add a New File” and either type a file name or use the “Browse” button to choose one. Then click “Submit”. You can submit a new version of a file later (before the deadline, of course).

A good strategy is to begin submitting the parts of your assignment that work early, and keep submitting improved versions as the deadline approaches. Only the last version of each file is graded.

appendix: more about minimax
using subtract square example

Minimax is a strong strategy that assumes that both players (let’s call them A and B) have all the information and time they need to make the strongest possible move from any game state (aka position). In order to choose the strongest possible move, players need a way to evaluate the best possible outcome, called the player’s score, they can guarantee from each position.

If the game is over, A’s score is:

- 1 if A wins
- -1 if A loses
- 0 if it’s a tie

B’s score is -1 times A’s score, since this is a zero-sum game: what’s good for A is bad for B. Suppose the game is not over, and the current player is A. Then A’s score is

- 1, if at least one A’s legal moves leads to a new position where A’s score is 1 (or B’s score is -1, since B is the current player in the new position)
- -1, if every legal move for A leads to a position where A’s score is -1 (or B’s score is 1, since B is the current player in the new position)
- 0, if at least one of A’s legal moves leads to a score of 0, but none lead to a score of 1.
This circular-sounding scoring process is guaranteed to provide an answer, assuming every sequence of moves eventually leads to the game ending. Indeed, it has a recursive solution, where the base case(s) corresponds to the end of the game.

Here’s a diagram representing all sequences of moves for a game of Subtract Square, starting with value being 6 and current player A, so that position is labelled \( A:6 \). Leading out from that position are lines (edges) labelled with the possible moves, and then positions those moves lead to... and so on.

We have labelled the positions where the game is over with A’s score on the left, and B’s score on the right. Work up from those positions, writing A’s and B’s scores beside each position, until you reach the top.

You were on the right track if you ended up with a score of 1 for A at position \( A:6 \). You can score any position in the games we’re working with using this technique. Here is a recursive algorithm for determining the score for the current position if you are the next player (the one about to play):

- **If the game is over there are no moves available, your score is either 1, 0, or -1, depending on whether you win, tie, or lose.**

- **If the game is not over, there are legal moves available. Consider each available move, and the position it takes the game to. Determine your opponent’s score in the new position (after all, your opponent is the next player in the new position). Multiply that score by -1 to determine your score in that position.**

  Your highest possible score among all the new positions is your score for the current position.

Of course, you will also want to record the move that gets you the highest score, so you might as well bundle them together in some suitable data structure.

Notice that you and your opponent are each solving different instances of the same problem, what your score is. That’s what makes the process recursive.