Assignment 1
Due 6pm, Thursday June 3, 2004

You must attach the cover sheet posted on the website. On this cover page, sign the statement on plagiarism. Without the cover sheet and this signed statement your homework will not be marked.

Requirements for assignment solutions:

- Staple your assignment.
- Write neatly, or if you do not possess this skill, type your solutions.
- Explain your solution so that someone from outside the course could understand.
- Be concise. More pages does not mean more marks.

1. (20 marks) Consider the following statement: (1)

   "If a program has a syntax error, then the program will not compile."

   (a) (3 marks) Define the domain and predicates necessary to translate the statement into precise symbolic notation.
   (b) (5 marks) Translate (1) into precise symbolic notation.
   (c) (4 marks) Give the converse of (1) first in English, then in precise symbolic notation.
   (d) (4 marks) Give the contrapositive of (1) first in English, then in precise symbolic notation.
   (e) (4 marks) Give the contrapositive of your answer to 1c in precise symbolic notation.

2. (40 marks) Assume you are given the following predicate symbols and your domain is \( \mathbb{N} \), the set of natural numbers (we assume that \( 0 \in \mathbb{N} \)).

   \( g(x, y) \): \( x \) is greater than \( y \)
   \( e(x, y) \): \( x \) equals \( y \)
   \( \text{sum}(x, y, z) \): \( x + y = z \)
   \( \text{prod}(x, y, z) \): \( x \cdot y = z \)

   (5 marks each) Translate the following statements into idiomatic English:

   (a) \( \forall x \in \mathbb{N}, g(x, 0) \)
   (b) \( \forall x \in \mathbb{N}, \exists z \in \mathbb{N}, \text{prod}(z, 2, x) \)
   (c) \( \forall a \in \mathbb{N}, \forall b \in \mathbb{N}, \forall c \in \mathbb{N}, (g(x, y) \land g(y, z)) \rightarrow g(x, z) \)
   (d) \( \neg(\forall n \in \mathbb{N}, \exists m \in \mathbb{N}, g(m, n)) \)
2. continued...

(5 marks each) Translate the following English statements into precise symbolic notation. Only use the predicates and domain defined above. Make sure you specify the domain of your variables in your solution and that your predicates are boolean.

(e) Every positive multiple of 5 is greater than 7.
(f) There is an odd integer.
(g) If \( x + y = z \) then \( y + x = z \).
(h) Not all integers are multiples of 2.

3. (5 marks each) Logic Puzzle: There are many brain teasers involving deserted islands and the people who inhabit them. One such puzzle, involves an island consisting of two different races. The Truth Tellers and the Liars. The Truth Tellers always tell the truth and the Liars, falsehoods. Suppose you meet three people \( U, V \) and \( W \) from this island. The first person \( U \) does not speak your language however \( V \) offers to translate. For each case, determine (if possible) from their responses to the following question, which race they each belong. If it is not possible, clearly show why it is not possible to determine which race at least one of \( U, V \) or \( W \) belong to.

How many of you are Truth Tellers?

Responses:

(a) \( V \): “\( U \) said, ‘Exactly one of us is a Truth Teller.'”
\( W \): “Don’t believe \( V \). He is a Liar”.
(b) \( V \): “\( U \) said, ‘Exactly one of us is a Liar.'”
\( W \): “\( V \)’s statement is true.”

4. (20 marks) Consider our example from class about rainy days.

(2) Every rainy day I bring an umbrella.
(3) If I bring an umbrella, then I stay dry.

(a) (3 marks each) For each of the following statements, determine whether it has the same meaning as (2). If it has a different meaning, make a small alteration to the statement so that it has the same meaning.

i. I bring an umbrella, if it is a rainy day.
ii. If it is a rainy day, I bring an umbrella.
iii. I bring an umbrella only if it is a rainy day.
iv. A rainy day is sufficient for me to bring an umbrella.
v. A rainy day is necessary for me to bring an umbrella.

(b) (3 marks) Assume that it is a rainy day. What conclusions can you draw given statements (2) and (3). Explain your reasoning.

(c) (2 marks) Now assume that I forgot my umbrella. What conclusions can you draw? Explain your reasoning.
5. (10 marks) Recall the table of hockey stats from class.

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</table>

(a) (4 marks) Draw a Venn diagram with sets that show “players with 5 or more goals”, “players who have played at least 11 games”, “players who have more points than games played”. Using the information in the table, enter each player’s number into the appropriate region of the diagram.

(b) (2 marks) Make a copy of the diagram in (5a) and shade in the region that corresponds to the statement “All players who have played less than 11 games yet scored more points than games played”.

(c) (2 marks) Make another copy of the diagram in (5a) and shade in the region that corresponds to the statement “Every player that has more points than games played and has scored at least 5 goals”.

(d) (2 marks) For each of (5b) and (5c) use the diagram to determine whether the statements are true or false.