CSC 258 Assignment 1, Fall 2008
Due by 5:00 p.m., Friday October 3, 2008; no late assignments without written explanation.
Please re-read the statement about collaboration versus plagiarism on the course info sheet!

1. In lecture, I gave an example of a light controlled by two roommates who each have their own switch. In one version the light was on only if both wanted it to be (the AND example); in another version the light was on if either wanted it to be (the OR example).

   Draw a circuit to implement the following slightly more complex rule. Each of the two roommates has an alarm clock, which has a logic output line telling you whether the alarm is sounding (ringing). (This happens in the morning.) They each also have a switch indicating that they wish to read in bed (this happens in the evening).

   Your circuit will have the above four inputs, and a logic output stating whether the overhead light should be on. For reading, the overhead light should be on only if both want it on; but when the alarm sounds, the overhead light should be on if either alarm is sounding.

2. Consider the following function:

   \[
   \begin{array}{cccc|c}
   w & x & y & z & F \\
   0 & 0 & 0 & 0 & 1 \\
   0 & 0 & 0 & 1 & 1 \\
   0 & 0 & 1 & 0 & 0 \\
   0 & 0 & 1 & 1 & 1 \\
   0 & 1 & 0 & 0 & 1 \\
   0 & 1 & 0 & 1 & 0 \\
   0 & 1 & 1 & 0 & 0 \\
   0 & 1 & 1 & 1 & 1 \\
   1 & 0 & 0 & 0 & 0 \\
   1 & 0 & 0 & 1 & 0 \\
   1 & 0 & 1 & 0 & 0 \\
   1 & 0 & 1 & 1 & 1 \\
   1 & 1 & 0 & 0 & 0 \\
   1 & 1 & 0 & 1 & 0 \\
   1 & 1 & 1 & 0 & 1 \\
   1 & 1 & 1 & 1 & 1 \\
   \end{array}
   \]

   (a) Derive a minimal sum-of-products expression for F using a Karnaugh map (show your Karnaugh map and show the algebraic expression).

   (b) Draw a logic gate diagram for this minimal sum-of-products expression.

   (c) If you could change any one function value (one of the 16 in the table above) to a “don’t care”, which change would minimize the minimal sum-of-products expression the farthest?

   (d) Draw the Karnaugh map corresponding to the change suggested in your answer to part c.

3. For each of the following equations, either prove (algebraically) that the equation is valid (always true), or disprove it by finding a counterexample (a set of assignments of values to variables which result in the equation being false).

   (a) \(a \oplus b = (a + b) \oplus (\bar{a} + \bar{b})\)

   (b) \(a \oplus b \oplus c = abc + a\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}\bar{b}c\)

   (c) \(a + (b \oplus c) = (a + b) \oplus (a + c)\)

(over)
4. This question involves working with a bizarre gate technology with only one gate: the BUT NOT gate. For inputs $x$ and $y$, this gate computes the function $x \overline{y}$.

![BUT NOT gate diagram]

Here is an example of working with the BUT NOT gate, henceforth abbreviated BN.

Suppose we want to implement $x + y$.

First note that for any $z$, $\overline{z}$ can be computed as $1 \text{ BN } z$.

Now,

$$x + y = \overline{x \overline{y}} \quad (\text{deMorgan's})$$
$$= \overline{x} \text{ BN } \overline{y}$$
$$= (\overline{1 \text{ BN } x}) \text{ BN } y$$
$$= 1 \text{ BN } ((\overline{1 \text{ BN } x}) \text{ BN } y)$$

So we can draw a circuit for $x + y$ using only BN:

![Circuit diagram for x + y using only BN]

The assignment question:
Draw circuit diagrams using only the BN gate (you may not use inverter bubbles, either) to compute the functions:

(a) $a + b + c$
(b) $a \oplus b$
(c) $a \oplus b \oplus c$

Show your algebraic work where applicable.

**Remember:**

This assignment is due at 5:00 p.m. on Friday October 3, 2008 (in the drop-box in BA 2220). Late assignments are not ordinarily accepted and always require a written explanation. If you are not finished your assignment by the submission deadline, you should just submit what you have, for partial marks.

Despite the above, I’d like to be clear that if there is a legitimate reason for lateness, please do submit your assignment late and send me that written explanation.