CSC 236 midterm

29 October 2004, 10:00, CG 150

Name (underline surname):

Student number:

Please circle your tutorial section:

F 10, surname A-F: Tovi Grossman, BA 3008

F 10, surname G-L: Dennis Kao, BA 3012

F 10, surname M-S: Han Liu, BA 3116

F 10, surname T-Z: Zhao Dan, BA B025

Aids permitted: One double-sided 8½x11" piece of paper. **Calculators are not permitted.**

Total: 30 marks.
Time allotted: 45 minutes.

Since time is short, be careful not to get stuck on one question to the exclusion of others. The amount of marks or answer-space allotted does not indicate how long it will take you to complete the question, nor does the size of the answer-space indicate the size of the correct answer.

Answer *all* questions. Answer questions in the space provided.

Do not open this booklet until you are instructed to.

Do not write anything in the following table:

	value	mark
1	10	
2	10	
3	10	
total	30	

1.

a) Prove that for all $n \in \mathrm{N}$, $\sum_{i=0}^{n} 2^{-i} = 2 - 2^{-n}$.

b) Prove that the following program is partially correct (i.e. you do not need to prove termination).

Precondition: $n \geq 0$
Postcondition: $x = n^2 - n + 5$
Loop invariant: $x = i^2 - i + 5$

```
x := 5
i := 0
while i < n do
    x := x + i + i
    i := i + 1
end while
```

2. The Fibonacci numbers can be defined as follows:

$F(0) = 0$
$F(1) = 1$
For $i \in Z$, where $i \geq 2$, $F(i) = F(i-1) + F(i-2)$

(Or we can start with F(1)=F(2)=1. Of course F(2) is 1 by either definition.)

The following program computes the nth Fibonacci number, where all variables are of type `int`:

```
i := 1
thisnum := 1
lastnum := 0
while i < n do
    nextnum := thisnum + lastnum
    lastnum := thisnum
    thisnum := nextnum
    i := i + 1
end while
```

a) Using "F(i)" to indicate the ith Fibonacci number, specify a precondition and a postcondition for this program.

Precondition:

Postcondition:

b) Using "F(i)" to indicate the ith Fibonacci number, specify a loop invariant for the 'while' loop.

c) We can prove the termination of this program by observing certain crucial facts about the value of $n - i$. State any two of them.

3. This question again discusses the Fibonacci numbers, where (as stated in question 2),

$F(0) = 0$
$F(1) = 1$
For $i \in Z$, where $i \geq 2$, $F(i) = F(i-1) + F(i-2)$

The beginning of the sequence looks like this:
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...

Note that every *third* element of the sequence is even.

Use induction to prove this, i.e. that $F(i)$ is a multiple of 2 iff i is a multiple of 3.